分类号 ______________ 密级 公开

UDC ______________ 编号 800212006b05

中国科学院研究生院
博士学位论文

辛方法在天体力学中的若干问题研究

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申请学位级别 博士 学科专业名称 天体测量与天体力学

论文提交日期 2006年5月 论文答辩日期 2006年6月

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答辩委员会主席 ______________

Typeset by $\LaTeX\,2_\varepsilon$ at June 30, 2006

With package `CASthesis` v0.1e of CTEX.ORG

The Study of Several Problems on Symplectic Methods in Celestial Mechanics

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June, 2006

Submitted in total fulfilment of the requirements for the degree of Ph.D. in Astrometry and Celestial Mechanics

摘　要

辛方法是一种求解哈密顿系统的数值方法。它具有保持哈密顿系统的辛结构和能量并具有高效稳定的优点。因而成为当前太阳系天体动力学定性演化的最佳积分工具。本学位论文对辛方法的几个问题作了理论和数值方面的探索。出于计算效率和数值精度的考虑，本文提出了一个赝三阶辛积分器。另一方面，哈密顿系统一般存在多个孤立积分，但辛方法除对应一个更改形式的能量积分外通常不能保持其它孤立积分。梯形公式作为一种特殊的辛方法却能保持Kepler问题中的Runge-Lenz 向量，本文对其进行系统详尽研究。此外，数值稳定性是数值方法理论研究的一个重要课题。为此，本文对几类辛方法的数值稳定性作了深入分析，找到了稳定区。具体表现在以下三个方面：

(1) **一个赝三阶辛积分器的构造**

对于可积分离的 Hamilton 系统 $H = H_0 + \sum_{i=1}^{N} \epsilon_i H_i(\epsilon_i \ll 1)$, 本文构造了一个赝三阶辛积分器。以日木土三体问题为物理模型数值实验表明该赝三阶辛积分器大约相当于 Wisdom-Holman 二阶辛积分器的一次校正或 Forest-Ruth 四阶辛算法的精度。此外，含力梯度的辛算法也适合处理 Hamilton 系统 $H = H_0(\mathbf{q}, \mathbf{p}) + \epsilon H_1(\mathbf{q})$, 其精度好于原辛积分器，但不优越于相应赝高阶辛积分器。

(2) **关于保持Runge-Lenz 向量的数值方法研究**

对孤立积分和能够保持 Runge-Lenz 向量的梯形公式进行详尽讨论。孤立积分就是限制粒子运动区域的不变量。具有 n 个自由度的自治可积 Hamilton 系统且只有 n 个互相对合的独立孤立积分, 并且其他孤立积分的存在对粒子的运动是有意义的。Kepler 二体系统存在能量积分、角动量积分和 Runge-Lenz 向量。对于平面运动情况，这三类积分中只有 3 个独立孤立积分；而对于三维空间情形，该三类积分仅有 5 个是独立的。就前者而言，Kepler 二体平面运动积分构成该系统中的对称群 $SO(3)$, 经过 Levi-Civita 变换，它可以转化为二维各向同性谐振子系统中的对称群，而该对称群能够被梯形公式准确保持。另一方面，对于后者梯形公式对这三类积分的严格保持还可以在 5 个 Kepler 轨道根数 a、e、i、Ω 和 ω 上得到体现。

(3) **几类辛方法的数值稳定性研究**

主要对1阶隐式Euler辛方法, 2阶隐式Euler中点辛方法, 1阶显辛Euler方法和2阶leapfrog显辛积分器共4种辛方法及一些组合算法进行了通常意义下的线性稳定性分析.针对线性哈密顿系统，理论上找到每个数值方法的稳定区，然后用数值方法检验了其正确性. 对于哈密顿函数为实对称二次型的情况，为了理论推导便利特推荐采用相似变换将二次型的矩阵对角化来研究辛方法的线性稳定性. 当哈密顿分解为主要部分和小摄动次要部分且二者皆可积时，无论是线性还是非线性系统，这种主次分解与哈密顿具有动势能分解的来比显著扩大了辛方法的稳定步长范围. 此外，对Mclachlan等提出的辛方法稳定性进行了讨论.

总之，本学位论文的几项工作具有一定的理论意义和应用价值。理论分析和数值探索指出针对哈密顿系统可以分解为一个主要部分和几个次要部分并且每部分均可积的情况，膺三阶辛积分器既具有较高的效率又有较好的数值精度，因而在实算中值得推荐。同时发现梯形公式能够严格保持各向同性线性哈密顿系统以及非线性Kepler问题的所有孤立积分，并且还准确保持了5 个 Kepler 轨道根数 a、e、i、Ω 和 ω。最后给出了几类辛方法的数值稳定区，特别揭示了Wisdom-Holman 2 阶 leapfrog 辛积分器当作用于一个可分解为一个主要部分和一个小摄动次要部分且二者皆可积的 Hamilton后不仅提高了数值精度而且大大扩展了稳定区。

关键词： 天体力学, 数值积分, 辛积分器, 孤立积分, Runge-Lenz 向量, 线性稳定性

Abstract

A symplectic method is a numerical method for the computation of Hamiltonian systems. Preserving the symplectic structure and energy, and having a high speed of calculation and good numerical stability, it has become the best choice in the study of the long-term qualitative evolution of Hamiltonian systems in solar system dynamics. This thesis deals with several problems on symplectic schemes by means of analytical and numerical methods. First, a pseudo-third-order symplectic integrator is constructed for the sake of a high speed of calculation and good numerical accuracy. Then, in detail we discuss the trapezoidal rule as one special symplectic integrator, which can conserve the Runge-Lenz vector of the Kepler problem. Finally, an analysis of the numerical stability for several symplectic integrators is given. More details are offered as follows.

(1) **The construction of a pseudo-third-order symplectic integrator**

A symplectic integrator is viewed as promise of being a valuable tool in the numerical exploration of planetary and satellite n-body systems in the solar system dynamics. For a separable Hamiltonian system of the form $H = H_0 + \sum_{i=1}^{N} \epsilon_i H_i (\epsilon_i \ll 1)$, we construct a pseudo-third-order symplectic integrator, which is shown to be almost equivalent to the first order corrector of the second symplectic integrator or the fourth order symplectic integrator of Forest and Ruth. In addition, symplectic integrator with force gradients can be applied to the Hamiltonian in the manner $H = H_0(\mathbf{q}, \mathbf{p}) + \epsilon H_1(\mathbf{q})$. As to the precision, the modified symplectic integrator is superior to the original one but no more than the corresponding pseudo-high-order one.

(2) **The study of a numerical method for the preservation of the Runge-Lenz vector**

An intensive discussion is given here on an isolating integral and the trapezoidal rule applied to the preservation of the Runge-Lenz vector introduced by Minesaki and Nakamura. The isolating integral is an integral of a dynamical system which can further restrict the motion region of a particle in the system. It is well known that an integrable autonomous Hamiltonian system with n degrees of freedom must hold n independent isolating integrals in involution each other. If there exist other independent isolating integrals in this system, these isolating integrals have significance. It is clear that a bound Kepler problem contains energy integral, angular momentum integral and the Runge-Lenz vector. It is found that a symmetry group $SO(3)$ formed by three independent isolating integrals in a dynamical system of two-dimensional Kepler motion is to be identified with a group of two-dimensional isotropic harmonic oscillator derived from the original Kepler system in terms of Levi-Civita transformation. As a result, the group of the isotropic harmonic oscillator can be strictly conserved by the trapezoidal rule. In addition, the trapezoidal rule can preserve exactly five orbital elements a, e, i, Ω and ω in a three-dimensional Kepler motion with five independent isolating integrals.

(3) **A Survey of Numerical Stability of Several Kinds of Symplectic Integrators**

This part deals mainly with an analysis of the linear stability of several symplectic integrators for a linear Hamiltonian system, which involve the first-order implicit symplectic Euler scheme, the second-order implicit centered Euler difference scheme, the first-order explicit symplectic Euler scheme and the second-order explicit leapfrog symplectic integrator. Meantime, a stable region for each integrator is found. The fact is also checked by numerical tests. Especially for a system with a real symmetric quadratic form, a simpler way to study the numerical stability is to use diagonalizing transformations. As an emphasis, a rather larger stable time step of each algorithm is admissible for either a linear or nonlinear system with an integrable separation of one main piece and another petty piece rather than a kinetic energy and a potential energy. On the other hand, we discuss the stability of symplectic integrators proposed by Mclachlan et al.

In sum, I believe that these works in this thesis are of the theoretical and practical significance in some degree.

Keywords: celestial mechanics, numerical integration, symplectic integrator,