PLASMA DYNAMICS IN ION TAILS
—A DISCUSSION OF THE INTERACTION OF SOLAR WIND PLASMA WITH COMETARY PLASMA

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ABSTRACT

On the assumption that the distribution function of ions in a comet tail is nearly Maxwellian and the distribution function of electrons and protons of solar wind is Gaussian, the interaction between the solar wind plasma and the cometary plasma and its stability are discussed. Applying the Noordergraaf form of van Kampen's stability criterion and using the modern observational data in the computations, the results show that the mechanism discussed cannot give rise to the instability. Neither might the electron-ion transient instability initially occur. This indicates that the high acceleration and instability occurring within the comet tail should be produced by other mechanisms.

I. INTRODUCTION

A lot of theoretical problems are provided by ion tails (i.e. Type I comet tail) which are rich in structure and variations. As far as their formation and motions are concerned, there still has not been any conclusive theory, particularly in reference to the problem of the acceleration of the tail material which are typically $10^7$—$10^8$ in unit of the solar gravitational acceleration.

It was first suggested by Biermann that the acceleration of the comet tail plasma could be explained by the strong coupling between the solar wind plasma and the cometary plasma. But up to now, concerning the mechanism and style by which the momentum flux of solar wind is transferred to cometary ions, the points of view among scientists are different. One important view is that the momentum transfer could be accelerated by the instability excited in cometary tails. Hoyle and Harwit have examined the two-stream instability in the zero magnetic field configuration. The results obtained by them indicate that this kind of instability set up through a collective interaction process would easily fade.

The plasma dynamics in cometary tails is re-examined in this paper. The improvements involved are:

1. Hoyle and Harwit assumed a Gaussian distribution function for individual plasma component (including solar electrons and protons and cometary electrons and ions), whereas we take a nearly Maxwellian distribution for cometary ions.
They assumed that the cometary plasma density is lower than the solar wind plasma density. This is opposite to the modern results of observations. All the computations in this paper are based upon the modern observational data.

In this paper, a boundary layer between ion tail and solar wind is considered and it is assumed that there is a velocity shear occurring within the ion fluid therein.

With these improvements mentioned above, the instability problem is reconsidered.

II. DISTRIBUTION FUNCTION OF PLASMA COMPONENTS
AND CRITERION OF INSTABILITY

It is assumed that both magnetic-field lines of solar wind and cometary tail lie along a direction radial to the sun, and particles of these two plasmas stream along field lines. It is easy to show, as far as plasma stability is concerned, this situation is the same as that of the configuration without magnetic field. Take this direction as $z$-direction. A transitional boundary layer from cometary tail to solar wind is assumed to be in the $x$-direction, and a density gradient and velocity shear are allowed within. Its associated diamagnetic drift velocity is then in the $y$-direction. The aim of this paper is to discuss the interaction of solar wind plasma with cometary plasma in this boundary layer.

In this case, we can utilize Smith and Geiler's results to describe ion tail, viz., the distribution function of tail ions is taken to be locally nearly Maxwellian\textsuperscript{[12]}. In the earth-fixed coordinate system, this distribution function is

\[ f_\theta = n_\theta f_{\theta,i} = N_\theta e^{-ix} f_{\theta,i} = N_\theta e^{-ix} \left( \frac{1}{2\pi \nu^i_{th}} \right)^{3/2} \exp \left\{ -\frac{v_x^2 + (v_y + \lambda \nu^i_{th}/\Omega)^2 + (v_z - \dot{\nu} - \gamma(x + v_y/\Omega))^2}{2
\nu^i_{th}} \right\}, \]

in which $\dot{\nu} = V_i + \lambda \nu^i_{th}/\Omega$, $V_i$ denotes the ion velocity of cometary tail, $\nu^i_{th}$ and $\Omega$ are the thermal velocity and cyclotron frequency respectively. The equilibrium is characterized by density gradient $\lambda$ and velocity shear $\gamma$:

\[ \begin{align*}
    n_\theta &= N_\theta e^{-ix}, \\
    v_{\theta y} &= -\lambda \nu^i_{th}/\Omega, \\
    v_{\theta z} &= \gamma(x - \nu^i_{th}/\Omega) + \dot{\nu} = V_i + \gamma x.
\end{align*} \]

Evidently, $n_\theta$ denotes the number density of ions of cometary tail per unit volume, $v_\theta$ the diamagnetic drift velocity. This form (2) of the distribution function we assumed guarantees that the velocity of cometary tail ions located on the inner side of the boundary layer is the velocity $V_i$ of ion tail referred to the earth.

Comet tail spectra show that only ions are present in the tail. This allows us to ignore tail electron in considering the problem. On the other hand, it is possible that the solar wind electrons and protons could move across the magnetic shielding and mix with ions in the boundary layer. We assume that each of these two plasma components obeys the Gaussian distribution. Apart from $V_n$ which denotes the velocity of solar wind with respect to the earth, all the notations are the same as in [4]. Then the normalized
velocity distribution functions of solar particles in the equilibrium state are

\[ f_{\text{e},x} = \left( \frac{2 \pi k T_{\text{e}}}{m_e} \right)^{-1/2} \exp \left[ - \frac{m_e (v - v_{\text{e}})^2}{2 k T_{\text{e}}} \right], \quad (3) \]

\[ f_{\text{p},x} = \left( \frac{2 \pi k T_{\text{p}}}{m_p} \right)^{-1/2} \exp \left[ - \frac{m_p (v - v_{\text{p}})^2}{2 k T_{\text{p}}} \right]. \quad (4) \]

From (1), corresponding to ions of the tail, we have

\[ f_{\text{i},x} = \left( \frac{1}{2 \pi v_{\text{th}}^2} \right)^{1/2} \exp \left\{ - \frac{v_x^2 + (v_y + \lambda v_{\text{th}}^2/Q)^2 + (v_z - \dot{\phi} - \tau (x + v_y/Q))^2}{2 v_{\text{th}}^2} \right\}, \quad (5) \]

in which \( k \) denotes Boltzmann constant, \( m_e, m_p \) the mass of electron and proton, and \( T_{\text{e}}, T_{\text{p}} \) the electron temperature and proton temperature of the solar wind.

In the following, write the component \( v_x \) along the \( z \)-direction of \( v \) as \( v \). Noticing

\[ \int_{-\infty}^{+\infty} f_{\text{e},x} dv_x dv_y = \frac{1}{\sqrt{2 \pi v_{\text{th}}}} \cdot \frac{1}{\sqrt{1 + \gamma^2/Q^2}} e^{-Q (v_{\text{th}}^2)}, \quad (6) \]

where

\[ C = \frac{Q^2}{Q^2 + \gamma^2} (v - V_i - \tau x)^2, \quad (7) \]

one can obtain

\[ F_{\text{e}}(v) = \int_{-\infty}^{+\infty} f_{\text{e}}(v) dv_x dv_y \]

\[ = \int_{-\infty}^{+\infty} \left[ \Sigma \Sigma f_{\text{e},x} \right] dv_x dv_y \]

\[ = \omega_{\text{te}}^2 \left( \frac{2 \pi k T_{\text{e}}}{m_e} \right)^{-1/2} \exp \left[ - \frac{m_e (v - V_{\text{e}})^2}{2 k T_{\text{e}}} \right] \]

\[ + \omega_{\text{tp}}^2 \left( \frac{2 \pi k T_{\text{p}}}{m_p} \right)^{-1/2} \exp \left[ - \frac{m_p (v - V_{\text{p}})^2}{2 k T_{\text{p}}} \right] \]

\[ + \omega_{\text{tr}}^2 \frac{1}{\sqrt{2 \pi v_{\text{th}}}} \cdot \frac{1}{\sqrt{1 + \gamma^2/Q^2}} e^{-Q (v_{\text{th}}^2)}, \quad (8) \]

in which the plasma frequency

\[ \omega_{\text{te}}^2 = \frac{4 \pi n_e e^2}{m_e}, \quad \omega_{\text{tp}}^2 = \frac{4 \pi n_p e^2}{m_p}, \quad \omega_{\text{tr}}^2 = \frac{4 \pi N_i e^2}{m_i} e^{-2\tau}. \quad (9) \]

and \( n_e, n_p \) and \( N_i e^{-2\tau} \) denote separately the number of electron, proton and ion per unit volume.

Let

\[ \omega_{\text{tr}}^2 \frac{1}{\sqrt{2 \pi v_{\text{th}}}} \cdot \frac{1}{\sqrt{1 + \gamma^2/Q^2}} e^{-Q (v_{\text{th}}^2)}, \]

\[ (8) \]

\[ \omega_{\text{te}}^2 = \frac{4 \pi n_e e^2}{m_e}, \quad \omega_{\text{tp}}^2 = \frac{4 \pi n_p e^2}{m_p}, \quad \omega_{\text{tr}}^2 = \frac{4 \pi N_i e^2}{m_i} e^{-2\tau}. \quad (9) \]
\[ \xi = \frac{v}{V_0}, \quad \alpha_i = \left( \frac{kT_e}{m_e} \right)^{2/3} V_e^{-1}, \quad \alpha_s = \left( \frac{kT_e}{m_e} \right)^{2/3} V_e^{-1}, \quad \alpha_s = \left( \frac{kT_e}{m_e} \right)^{2/3} V_e^{-1}, \]

\[\sigma = \frac{V_i + \gamma \xi}{V_0} = \frac{v_{\infty}}{V_e},\]

the meaning of \(\sigma\) is obvious. Then

\[ F_i(v) = F_i(V_0 \xi) = F(\xi) \]

\[ = \frac{1}{V_0 \sqrt{2\pi}} \left[ \frac{\omega_{n_e}}{\alpha_i} \exp \left( -\frac{(\xi - 1)^2}{2\alpha_i^2} \right) + \frac{\omega_{i_e}}{\alpha_i} \exp \left( -\frac{(\xi - 1)^2}{2\alpha_i^2} \right) \right. \]

\[ + \left. \frac{\omega_{i_p}}{\alpha_i} \exp \left( -\frac{(\xi - \sigma)^2}{2\alpha_i^2} \right) \right] \cdot \alpha_i \]

Taking derivative according to \(\xi\), we have

\[ V_0 \sqrt{2\pi} F'(\xi) = -\left[ \frac{\omega_{n_e}^2}{\alpha_i^3} (\xi - 1) \exp \left( -\frac{(\xi - 1)^2}{2\alpha_i^2} \right) + \frac{\omega_{i_e}^2}{\alpha_i^3} (\xi - 1) \exp \left( -\frac{(\xi - 1)^2}{2\alpha_i^2} \right) \right. \]

\[ - \left. \frac{(\xi - 1)^2}{2\alpha_i^2} + \frac{\omega_{i_p}^2}{\alpha_i^3} (\xi - \sigma) \exp \left( -\frac{(\xi - \sigma)^2}{2\alpha_i^2} \right) \right]. \]

Hence

\[ V_0 \sqrt{2\pi} F''(\xi) = -\left[ \frac{\omega_{n_e}^2}{\alpha_i^3} \left[ 1 - \frac{(\xi - 1)^2}{2\alpha_i^2} \right] \exp \left( -\frac{(\xi - 1)^2}{2\alpha_i^2} \right) \right. \]

\[ + \left. \frac{\omega_{i_e}^2}{\alpha_i^3} \left[ 1 - \frac{(\xi - 1)^2}{2\alpha_i^2} \right] \exp \left( -\frac{(\xi - 1)^2}{2\alpha_i^2} \right) \right] \]

\[ + \left. \frac{\omega_{i_p}^2}{\alpha_i^3} \left[ 1 - \frac{(\xi - \sigma)^2}{2\alpha_i^2} \right] \exp \left( -\frac{(\xi - \sigma)^2}{2\alpha_i^2} \right) \right]. \]

When \(\xi = \xi_0\), if \(F(\xi_0)\) has a minimum, namely, \(F'(\xi_0) = 0, F''(\xi_0) > 0\), the instability criterion then reads

\[ U(\xi_0) = P \int_{-\infty}^{\xi_0} \frac{F'(\xi)}{\xi - \xi_0} d\xi \]

\[ = -\frac{1}{V_0} \left[ \frac{\omega_{n_e}^2}{\alpha_i^3} + \frac{\omega_{i_e}^2}{\alpha_i^3} \left( \frac{\xi_0 - 1}{\alpha_i} \right) + \frac{\omega_{i_p}^2}{\alpha_i^3} \left( \frac{\xi_0 - \sigma}{\alpha_i} \right) \right] > 0. \]

in which \(P\) denotes the principal value and

\[ h(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi_0} \frac{x}{e^{x^2/2}} dx. \]

### III. Computational Results and Discussion

According to the physical observations of comets in recent years, we use the following reasonable data in computations, viz., take \(T_e = 10^9 K, T_i = 4 \times 10^4 K, T_i = 10^5 K, \)
\[ n_e = n_p = 5, \quad N_e = 100, \quad V_s = 540 \text{ km/sec}, \quad V_i = 250 \text{ km/sec}. \]

The thermal velocity \( v_{th} \) of ions is obtained as,

\[ v_{th} = \left( \frac{2kT_i}{m_i} \right)^{1/2}, \]

where \( m_i = 28 \text{ amu} \). This is because CO\(^+\) band is always the strongest in the spectra of ion tails and the emission of N\(_2\)^+, although weaker than that of CO\(^+\), has its brightness distribution similar to that of CO\(^+\) and extends far into the cometary tail. They are the two dominant ions in the tail with the same mass and charge, and we can treat them as a single component.

About the magnetic field of cometary tail, in their profound discussion of Alfvén model, Ip and Mendis estimate that a magnetic field of the order of 100 \( \gamma \) may be built up in tails\(^{[1]} \). The analysis of observational results of Kohoutek comet leads to the same conclusion\(^{[1]} \). Hence we take \( B_z = 10^{-3} \text{ Gauss} \) and have

\[ Q = \frac{|e|B_z}{m_i c} = 3.44 \times 10^{-1}. \]

Use \( L \) to denote the thickness of the boundary layer, then the velocity shear \( \gamma = (V_s - V_i)/L \). Taking \( L = 100 \text{ km} \), we get \( \gamma = 2.9 \). We compute for two cases: \( \lambda = 3 \times 10^{-7} \) and \( \lambda = 4.6 \times 10^{-7} \), corresponding to the cases that the ion number density of tail in the boundary layer decreases from 100 cm\(^{-3}\) at the inner boundary to 5 and to 1 at the outer boundary respectively. The values of \( x \) and their corresponding \( \sigma \) are taken as shown in Table 1.

Each calculated \( F(\xi) \) has no minimum, but has a maximum near \( \xi = 1 \). This indicates that the instability of plasma cannot occur, neither does that kind of transient electron-ion instability as suggested by Hoyle and Harwit. The instability phenomena observed in ion tail might be caused by some other mechanisms associated with magnetic

![Figure 1](image)

**Fig. 1.** The curve of function \( F(\xi) \) for the case of \( \lambda = 5 \times 10^{-3} \) and \( x = 50 \text{ km}. \)
field.

Fig. 1 is the curve of $F(\xi)$ corresponding to $\lambda = 3 \times 10^{-1}$, and $x = 50$ km. The peak-form structure near $\xi = 1$ is contributed by the term associated with solar proton. Fig. 1 is a characteristic diagram, and all of the other $F(\xi)$'s are similar to it.

REFERENCES