SCALING FROM JUPITER TO PULSARS AND MASS SPECTRUM OF PULSARS

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ABSTRACT

Following the principle of similitude, we scaled the energy loss of Jupiter through the acceleration of high energy particles to that of Crab pulsar. The remarkable agreement suggests a classification scheme for all pulsars in which we relate the angular velocity \( \omega \) and \( \left( -\frac{d\omega}{dt} \right) \) of a pulsar to its moment of inertia \( I \) divided by its radius \( R_0 \) and then determine its mass according to neutron star models. The mass spectrum of 194 pulsars, \( dN/dm \), is found to be \( \sim 1/m^2 \).

Subject headings: planets: Jupiter — pulsars — stars: rotation

1. INTRODUCTION

One of the important findings by space experiments is the presence of suprathermal particles in active plasma media. The initial surprise at the discovery of the Earth’s radiation belt has now become an expectation of the detection of energetic particles in every active medium. Apparently, so long as there is an energy source, there will be energetic particles. The mechanisms for the acceleration appear to be rather complicated; they apparently involve plasma instability processes of various kinds which are not well understood, even qualitatively. However, among all the systems studied, Jupiter’s magnetosphere seems to be the simplest, because it has a strong magnetic field of its own and is sufficiently far away from the Sun to make the effect of the solar wind less serious. Furthermore, Jupiter does not have extended rings like Saturn which complicate the dynamics of the radiation belt. Thus, Jupiter can be regarded as a cold spinning magnetized body surrounded by a low-density plasma medium. In this respect, it resembles a pulsar.

The resemblance between Jupiter and a pulsar was first suggested by Burbidge and Strittmatter (1968) from consideration of the characteristics of the radio noise emitted by the two objects. The interest in the analogy was rekindled by the observation on the structure of Jupiter’s outer magnetosphere on Pioneer 10 and Pioneer 11 (Kennel and Coroniti 1975; Michel 1979). After the encounters of Voyager 1 and Voyager 2 with Jupiter, the information on the particle population over a wide energy range in the magnetosphere and their flow direction became available. In this paper, we make use of this information for a quantitative comparison of the energy generation by Jupiter with that by a pulsar and then explore the consequence of the scaling.

The first indication of the existence of intense energetic particle fluxes within Jupiter’s environment was inferred from its decameter radiation (Burke and Franklin 1955). Direct measurements were first made with instruments on Pioneer 10 and subsequently on Pioneer 11, Voyager 1, and Voyager 2. A feature of the radiation environment relevant to the present paper is the observation of a large flux of high-energy particles escaping from the Jovian magnetospheric region (Krimigis et al. 1979; Krimigis et al. 1981a). An order of magnitude estimate indicated that the total energy needed to maintain the intensity level is about \( 2 \times 10^{30} \)–\( 2 \times 10^{31} \) ergs s\(^{-1}\) (Krimigis et al. 1981a). An estimate from Jovian auroral emission arrived at a value of the same order of magnitude (Broadfoot et al. 1979). Assuming a reasonable conversion efficiency, the solar wind can supply only a fraction of this energy. It was thus concluded that the energy must be deduced from the rotational energy of Jupiter. In other words, particles of the Jovian environment are accelerated at the expense of its rotational energy, and this energy is eventually lost to space. As far as Jupiter is concerned, the rotational energy and the angular momentum carried away by the escaping particles are extremely small. However, the essence of the finding is that it provides a model of the mechanism by which a fast rotating massive celestial body, such as a T Tauri star or a neutron star, may lose its energy and angular momentum in the course of its evolution.

In order to scale the results on Jupiter to pulsars, one

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is confronted with two tasks: the first is to derive an expression for the rate of acceleration of energetic particles by a rotating magnetic body, and the second is to obtain an expression for the magnetic dipole moment of the body. In view of the complexity of the particle acceleration mechanisms and an equally or even more complex nature of the magnetism of celestial bodies, one can only hope to make order of magnitude estimates. One approach is to use dimensional analysis (known also as the principle of similitude, see Rayleigh 1915). It turns out that, by using the observational results on Jupiter and other planets as a guide, one can obtain an expression which agrees remarkably well with the observational results on the Crab pulsar. The same expression, in conjunction with neutron star models, also allows one to determine the mass of a pulsar whose angular velocity \( \omega \) and \( (\omega/|\omega|) dt \) are known. We have determined the masses of 194 pulsars and found a mass differential spectrum as \( 1/m^2 \). A preliminary version of this paper was reported at the 17th International Cosmic Ray Conference (Fan and Wu 1981).

II. ACCELERATION OF ENERGETIC PARTICLES BY SPINNING MAGNETIZED BODY

a) The Principle of Similitude

The concept of similitude has been applied successfully to obtain “laws” of physics in many fields. A classical example is the universal equilibrium turbulence spectrum, termed by its originator Kolmogoroff (1941a, b, c) as the theory of local similarity. In his well-known essay on the principle of similitude, Rayleigh (1915) listed 15 physical applications for this method. In connection with the examples, he stated the following as the general steps in applying the principle: It is necessary as a preliminary step to specify all the quantities on which the desired result may reasonably be supposed to depend. Then, determine the combination of the dimensions of the various quantities that can give that of the final result. The numerical coefficient, if unique, must be found empirically (or by future theory).

b) Acceleration of Particles in a Magnetosphere

The acceleration of energetic particles in the vicinity of a spinning celestial body is due to electric fields generated in the medium surrounding the body. Assume that the magnetic dipole axis is aligned with the rotational axis. Then, the rate of energy generation would depend on only four physical quantities: the magnetic dipole moment \( M \), the angular velocity \( \omega \), the linear dimension of the body \( l \), and an empirical quantity \( K \) characterizing the physical nature of the plasma medium in its vicinity. Our objective is to find an expression which contains these four quantities and has the dimensions of energy per second.

The functional form can be readily obtained by the following considerations: The acceleration of particles must be by electric fields \( E \); if \( B \) be the magnetic field, \( B/\omega c \) has the dimension of \( E \), \( B \sim E/|B| \), and \( E^2 \) has the dimension of energy per unit volume. Thus, the rate of energy generation is

\[
\frac{dT}{dt} \sim K \left( \frac{B \omega}{c} \right)^2 \sim K \left( \frac{M}{l^3} \right)^2 \left( \frac{\omega}{c} \right)^2 \sim K \frac{M^2 \omega^2}{l^2 c^2},
\]

where \( c \) is the velocity of light and \( K \) is a numerical coefficient having the dimension of \( s^{-1} \) and a physical interpretation of being the “conductivity” of the plasma medium. From a rather obvious physical consideration, we shall use \( R_0 \), the equatorial distance beyond which the acceleration begins to take place, in the place of \( l \). Thus,

\[
\frac{dT}{dt} = K \frac{M^2 \omega^2}{R_0 c^2}.
\]

For a celestial body like Jupiter, \( R_0 \) is just the equatorial radius. On the other hand, for a body with an extended atmosphere, \( R_0 \) is then the radial extension of the upper atmosphere. The coefficient \( K \) is used to absorb all the constants except \( c^2 \).

The term \( K \) in equation (1) is a numerical coefficient characterizing the gross nature of the medium in the environment of the body and thus reflecting the efficiency of the acceleration. By equating equation (1) with the observed value of \( dT/\partial t \) for Jupiter, we determine the value of \( K \). The coefficient \( K \) is then regarded as the scaling constant for the energy output by rotating bodies like Jupiter; different classes of bodies assume different values of \( K \). The validity of such scaling must be determined by observation. It is interesting to note the parallelism between equation (1) and Ohm’s law \( J = \sigma E \). Like \( \sigma \), the constant \( K \) specifies the gross nature of the medium which intrinsically includes the temporal and spatial averages over all the fine details.

There are three published values of \( dT/\partial t \) for Jupiter, one from the observed flux of escaping particles from the Jovian magnetosphere, the second, from the mass loss and energy balance (Krimigis et al. 1981a), and the third, from Jovian auroral emission (Broadfoot et al. 1979). They range from \( \sim 2 \times 10^{-10} \) to \( 2 \times 10^{-21} \) ergs \( s^{-1} \). These values are within the range estimated by Fillius, Ip, and Knickerbocker (1977) from the energy content in escaping high-energy electrons observed on Pioneer 10 and Pioneer 11. From these estimates we have:

\[
K = 2 \times 10^{-2} \quad \text{and} \quad 2 \times 10^{-1} \quad \text{s}^{-1},
\]

respectively. An improved estimate for the value of \( K \) will be given below.

Note that, in deriving equation (1), no mention was made of the possible effect of Jupiter’s satellites such as Io. We assume that the material from Io and other satellites, together with that from Jupiter’s upper atmosphere and the solar wind, determine the composition of the surrounding plasma medium. Thus, any effect is folded into the efficiency coefficient \( K \).

III. MAGNETIC DIPOLE MOMENT—"BODE’S LAW"

Scientists have been fascinated for a long time by the apparent proportionality between the angular momen-
tum of a celestial body and its magnetic moment. As early as 1891, Schuster (1891), suspecting from the appearance of solar corona that the Sun might have a magnetic field, put forward the question, "Is every large rotating mass a magnet?" The first serious attempt to relate the two quantities was made by Wilson (1923) and later revised by Blackett (1947). They assumed that the angular momentum $L$ and the magnetic moment $M$ of a spinning body are linearly related by a universal constant of proportionality.

The magnetic field of a celestial body must be produced by an interior current system, and this current system is presumably excited by rotation (Levy 1976). Therefore, it seems reasonable to assume that, as a first approximation, the dipole moment depends linearly on its angular momentum with some numerical coefficient characterizing the excitation mechanism. This assumption is the so-called magnetic "Bode's law," giving the magnetic dipole moments of the planets.

The "law" can be seen in a plot of the logarithm of the magnetic dipole moments against the logarithm of the angular moments of several celestial bodies, this plot being roughly a straight line (Kennel 1973; Hill and Michel 1975; Dessler 1976; Siscoe 1978; Russell 1978; Busse 1978). Because the physical reason for the approximate linearity is not fully understood (a situation similar to the empirical rule on the proportionate distances of several planets from the Sun, discovered by Bode and announced by Titius in 1766), the name "Bode's law" is sometimes used.

In Fig. 1 we reproduce the plot by Russell (1978) of the magnetic moments versus angular momenta of seven celestial bodies, Mercury (My), Venus (V), Mars (Ms), Earth (E) and Earth-Moon system (E-M), Jupiter (J), Saturn (S), and the Sun, modified slightly according to more recent measurements. The solid straight line is drawn through the points for (E-M) and (J), whereas the dashed line is through the points (E) and (J). The

![Figure 1](image-url)

Fig. 1.—"Bode's Law" of planetary magnetism (Russell 1978). Magnetic moment $M$ normalized to terrestrial moment $M_E$ is plotted against angular momentum $L$ normalized to terrestrial angular momentum $L_E$. My, Mercury; V, Venus; Ms, Mars; E, Earth; E-M, Earth-Moon system; J, Jupiter; S, Saturn; Sun; NP 0532, Crab Pulsar (calculated according to eq. [2]).
angular momenta and the magnetic moments are all measured in units of that of the Earth. The equation for the solid straight line is:

\[ M = zL, \quad z = 2 \times 10^{-16} \text{ gauss cm s}^{-1}, \]  

(2)

where the magnetic dipole moment \( M \) is in units of gauss \( \text{cm}^3 \) and the angular momentum \( L \) is in \( \text{g cm}^2 \text{s}^{-1} \). The apparent linear relationship between the magnetic dipole moments and the angular momenta shown in Figure 1 suggests an approximate similarity in the magnetic field generation among these bodies, although the physical processes involved in the excitation of the current systems are undoubtedly complex. We shall assume that \( L \) and \( M \) for a given class of objects are linearly related and coefficient \( z \) is a scaling constant.

By combining equations (1) and (2), we arrive at the expression for the rate of energy output by a spinning celestial body as

\[ \frac{dT}{dt} = \frac{Kx^2}{c^2} L \omega^2 = \frac{Kx^2}{c^2} R_0^2 \omega^4, \]  

(3)

since \( L = IO_0 \), where \( I \) is the moment of inertia of the body. The quantity \( Kx^2 \) is regarded as a scaling constant. From the observations on Jupiter,

\[ Kx^2 = 8 \times 10^{-34} \times 8 \times 10^{-33} \text{ cm s}^{-1} \text{ g}^{-1}, \]

depending on the choice of the value of \( K \).

It is interesting to note that, in contrast to Jupiter’s offset dipole (tilted by 10° with respect to the rotational axis), the dipole of Saturn is practically centered and the angle between \( M \) and \( L \) is less than 1°. Yet, the Low Energy Particle Telescope (LEPT) on *Voyager 1* detected escaping ions from the Saturnian magnetosphere very much like those observed in the vicinity of Jupiter (Krimigis et al. 1981b). This observation indicates that the energy loss does not depend sensitively on the tilted angle, rendering support for equation (1). The observed escaping 53–80 keV ions is about two orders of magnitude lower than that for Jupiter, presumably due to the fact that the dipole moment of Saturn is \( \frac{1}{50} \) that of Jupiter. However, detailed comparison with that expected from equation (1) is not yet possible because (a) the particle fluxes in other energy ranges are not available at present, and (b) the effect of the rings on particles is not yet known.

Before applying equation (3) to the rate of energy generation by the Crab pulsar to demonstrate the scaling, we shall make a brief review of the magnetic dipole radiation model for the energy generation.

**IV. A BRIEF REVIEW OF THE THEORIES OF THE ENERGY GENERATION BY PULSARS**

After the discovery of pulsars in 1967–1968, Pacini (1968), Gold (1968), Gunn and Ostriker (1969), and Ostriker and Gunn (1969) proposed an interpretation of the phenomenon (see also Pacini 1967 before the discovery). They assumed that the energy loss of a pulsar can be regarded as a magnetic dipole radiation. From classical electrodynamics, one has

\[ \frac{dT}{dt} = -i \omega \frac{d\omega}{dt} = \frac{2}{3c^3} M^2 \omega^4, \]

(4)

for the radiation emission per second by a rotating magnetic dipole of dipole moment \( M \) and an angular velocity \( \omega \).

Given equation (4) for the magnetic dipole radiation, one can determine \( I \) and \( M \) of a pulsar if its energy loss \( dT/dt \) can be estimated. For the Crab pulsar (Trimble and Rees 1970),

\[ \frac{dT}{dt} \sim 3 \times 10^{38} \text{ ergs s}^{-1}, \]

which gives

\[ I = 6 \times 10^{44} \text{ g cm}^2 \quad \text{and} \quad M = 3 \times 10^{30} \text{ gauss cm}^3. \]

These values are of the same order of magnitude as that estimated from other considerations. For instance, the value of \( I \) is consistent with that of a neutron star of mass \( \sim 1.0 \text{ solar mass} \) and a radius \( \sim 10 \text{ km} \).

Further deduction from equation (4) was hampered by the fact that one does not know the dependence of \( M \) on other physical parameters such as \( \omega \). The proposed dependence by Ostriker and Gunn (1969), \( M^2 = M_0^2 e^{-\zeta \omega} \), still contains an unknown parameter \( \zeta \). However, if one assumes that \( M \) is a constant or \( \zeta \) is a very small quantity, then equation (4) suggests that

\[ \frac{-d\omega}{dt} \sim \omega^3. \]

In Figure 2 we plotted the measured values of \((-d\omega/dt)\) for 202 pulsars versus their \( \omega \)'s. The points do show a trend as \( \omega^3 \) with a spread which can be interpreted as the distribution of the values of \( M^2/I \).

An integration of equation (4) yields an expression for the dependence of \( \omega \) on \( t \). If \( \omega_0 \) is the initial velocity and

\[ \gamma = \frac{4M^2}{3c^2I} = -\frac{2}{\omega^3} \frac{d\omega}{dt}, \]

we have

\[ \frac{1}{\omega^2} - \frac{1}{\omega_0^2} = \gamma t \quad \text{or} \quad \omega = \omega_0 (1 + \omega_0^2 \gamma t)^{-1/2}. \]

(5)

For the Crab pulsar, \( \gamma = 7 \times 10^{-16} \text{ s} \), \( \omega_0 = 1.9\omega = 357 \text{ rad s}^{-1} \)

It can be shown from equation (4) that

\[ n = \omega \frac{d^2\omega}{dt^2} \left( \frac{d\omega}{dt} \right)^2 = 3 \]

and

\[ t_e = \frac{1}{\gamma \omega^2} = -\frac{\omega}{|d\omega|/dt} \]

(6)

as the upper limit of a pulsar’s age. For the Crab pulsar, the observed value of \( n \) is about 2.5 (Groth 1975). The
Goldreich and Julian (1969) elucidated the characteristics of the motion of charged particles in a pulsar’s magnetosphere, but they assumed that the magnetic fields are not distorted by the currents. Because of the strong fields, the charged particles can only flow along field lines, some of which are open to the light cylinder. In order to close the current “circuit,” a cross-field current system near the star surface was invoked (Ruderman and Sutherland 1975; Kennel et al. 1979). Then the spin-down rate of the pulsar can be obtained by either calculating the torque exerted on the pulsar by $J \times B$ force or by considering the Maxwell stresses over a surface surrounding the pulsar. They both lead to an expression for $dT/dt$ practically identical to equation (4). Two unanswered questions remain: (1) Can the contribution to the magnetic fields by the magnetospheric currents be neglected? (2) Why is equation (4) not applicable to the case of Jupiter?

In this paper, we take a different approach. We assume that the magnetic fields in a pulsar’s magnetosphere, especially in the outer region, is dominated by that from the currents, similar to what was observed on Jupiter, and the physical condition in the region is so complicated that one can only hope to use dimensional analysis to obtain an expression for the energy loss with one scaling constant to be determined experimentally. Note that $dT/dt$ in equation (1) depends on $\omega^5$, in contrast to the $\omega^6$ dependence in equation (4). It is the linear relationship between $M$ and $L$, equation (2), which retains the $\omega^6$ dependence of $dT/dt$ in equation (3).

V. SCALING FROM JUPITER TO THE CRAB PULSAR

Pulsars are perhaps the only class of celestial bodies which offer convincing evidence that they are accelerating high-energy particles in their environment at the expense of their rotational energies. Since the Crab pulsar (NP0532) is the most extensively studied of these objects, we shall use it as a test case for the idea developed in the foregoing sections.

Equation (3) gives the rate of energy generation by a rotating magnetized celestial body whose magnetic dipole moment $M$ is related to its angular momentum $L$ by $M = \alpha L$ and whose surrounding medium is characterized by an efficiency coefficient $\kappa$. The value of $K\sigma^2$ for Jupiter is $8 \times 10^{-34} \times 8 \times 10^{-32}$ (cm s$^{-1}$ g$^{-1}$). The physical meaning of scaling is to show that the value of $K\sigma^2$ deduced from the observation on the Crab pulsar has the same value as that for Jupiter.

In order to avoid the dependence on a specific neutron star model, we use $6 \times 10^{44}$ g cm$^2$ for the value of $I$ deduced from the observed $dT/dt$ by Trimble and Rees (1970) and $10^6$ cm for the value of $R_0$ which can be estimated from the argument of stability of a fast spinning celestial body. For the Crab pulsar, $\omega = 190$ rad s$^{-1}$, and $-\sigma/\sigma = 2.43 \times 10^{-9}$ s$^{-2}$. We find

$$K\sigma^2 = 5.3 \times 10^{-34} \text{ cm s}^{-1} \text{ g}^{-1}.$$
As compared to the lower value of $K \alpha^2$ deduced from the observations on Jupiter, $8 \times 10^{-34}$, the agreement is rather astonishing. We thus have demonstrated the justification for the scaling.

It may not be too surprising to the scaling of the energy generations, since (a) the nearby environment of Jupiter is not much disturbed by solar wind and solar radiation, and (b) Jupiter behaves like a cold star. On the other hand, the magnetic field of Jupiter is presumably excited by the internal dynamo action, while that of the Crab pulsar may be frozen-in; they are too different in origin to be compatible. One plausible explanation for the scaling is that the frozen-in field retains the nature of its parent star, which may not be too different from the Sun.

Equations (3) and (4) bear the same functional form in $\omega$ except that, unlike $M^2$ in equation (4), $I^2/R_0$ in equation (3) is truly a constant. Therefore, equation (3) explains naturally the $\omega^3$ dependence of $(-d\omega/dt)$ as exhibited in Figure 2 and yields the identical results as in equations (5) and (6).

VI. THE MASS SPECTRUM OF PULSARS

Having demonstrated the observational evidence for the scaling, we now proceed to link observational facts on pulsars to neutron star models, from which we obtain the mass spectrum of pulsars.

Equation (3) can be written as

$$-\frac{1}{\omega^2} \frac{2}{\omega} \frac{d\omega}{dt} = \frac{2}{\omega^3} \frac{I}{c^2} R_0 K \alpha^2.$$

For a pulsar, the quantity of the left-hand side can be precisely measured, whereas that on the right-hand side is determined by the mass of the pulsar according to neutron star models. $K \alpha^2$ is now regarded as a scaling constant for all pulsars and allowed to be adjusted to agree with neutron star mass limits.

To illustrate this scheme, we plotted in Figure 3 the measured value of $(2/\omega^3)(-d\omega/dt)$ versus the angular velocities $\omega$ of 202 pulsars whose $\omega$ and $d\omega/dt$ have been measured. On the right-hand ordinates we marked a new scale for $I/R_0$, which is

$$I = \frac{c^2}{2K \alpha^2} \left( -\frac{2}{\omega^3} \frac{d\omega}{dt} \right),$$

with the scaling constant $K \alpha^2$ taken as $8 \times 10^{-34}$ from the measurements on Jupiter. Then, for a given value of $I/R_0$, there corresponds a neutron star mass according to a certain neutron star model. Figure 4 shows the relation from six different models: (A) Pandharipande (1971a), pure neutron; (B) Pandharipande (1971b), pure neutron plus hyperon; (C) Bethe and Johnson (1974), Model I; (D) Bethe and Johnson (1974), Model V; (E) Moszkowski (1974), not shown because it coincides with (A); (F) Arponen (1972); and (G) Canuto and Chitre (1974) (see also Canuto 1978). As can be seen, as far as our purpose is concerned, they do not make much difference. From the figure, we determined the masses of 194 pulsars. The remarkable feature is that, without adjustment of the scaling constant $K \alpha^2$, the masses thus determined are confirmed within the neutron star mass limits ($\sim 0.1$–3 solar masses $M_\odot$) in complying with current neutron star models.

We then divided the mass range 0.08–3.0 $M_\odot$ into five bins, 0.08–0.15, 0.15–0.25, 0.25–0.5, 0.5–1.0, and 1.0–3.0 $M_\odot$. Using the numbers of pulsars in these bins, which are 107, 31, 29, 14, and 13, respectively, we computed the numbers of pulsars per 0.1 $M_\odot$ and plotted them as a function of $m/M_\odot$ in Figure 5. (The Crab pulsar has a mass 0.6 $M_\odot$.) The error bars in Figure 5 represent the statistical deviations in the pulsar numbers divided by the mass range in units of 0.1 $M_\odot$.
A least-squares fit yields a spectrum as

$$\frac{dN}{dm} \sim \frac{1}{m^x}$$  \hspace{1cm} (9)

with $x = 1.8$. Eight stars are excluded from the plot: they are \( \text{P0153+61} \) and \( \text{O820+02} \) whose \( I/R_0 \) values are \( \sim 10^{40} \), and \( \text{O254-53} \), \( \text{O151-52} \), \( \text{O1804-08} \), \( \text{O1913+16} \), \( \text{O1944+17} \), and \( \text{O1952+20} \) whose \( I/R_0 \) values are \( \sim 10^{38} \) g cm, which are beyond the limits of the theoretical curves in Figure 4 to allow the determination.

It is noted that \( \text{P1913+16} \) is a binary pulsar and has the smallest value for \( (2/\omega^3)(-d\omega/dt) \). From the precise measurements by Taylor et al. (1979) on the pulse arrival time, the mass of the pulsar was estimated as \( 1.4 M_\odot \). It contradicts the value which one may conclude from the plots in Figures 3 and 4. A plausible explanation for this disagreement is the coupling with its companion star which makes the assumption used in deriving equation (3) no longer valid. An accretion disk around the pulsar could also cause the absorption of the high-energy particles in its vicinity, thus reducing the value of \( (2/\omega^3)(-d\omega/dt) \). One interesting question arises as to whether the other five stars may also be binaries. In the upper mass region, according to the plots in Figures 3 and 4, \( \text{P0153+61} \) and \( \text{O820+02} \) are in the region where stars would collapse into black holes. One possibility is that the collapse will occur when their spin velocities are sufficiently reduced. The other possibility is that the scaling constant \( K\alpha^2 \) may have to be adjusted upward to reduce all the \( I/R_0 \) values by a factor of 3. Since this paper is of an exploratory nature, any fine adjustment is not justified.

VII. DISCUSSION

The basic assumption of this paper is that a pulsar is slowing down as the result of the acceleration of particles...
in its nearby plasma medium and the rate of energy generation is given by equation (1) deduced from dimensional argument. The linear relationship between the magnetic dipole moment $M$ and the angular momentum $L$ (equation [2]) is an extrapolation from "Bode's law" of the planetary magnetism on the one hand and is demanded by the $\omega^3$ dependence of $(-d\omega/df)$ of 202 pulsars (shown in Fig. 2) on the other. The results deduced from these two equations seem to agree rather well with all the observational results.

The formal relationship between $M$ and $L$ expressed in equation (2) is, of course, not new; one has seen it connected to the spin of an elementary particle. Instead of Blackett's ambitious attempt to relate $\alpha$ to universal constants (Blackett 1947), we may regard $\alpha$ as a coefficient characterizing the interior structure of the rotating body. Then equation (2) becomes a universal expression which may have a profound physical significance.

The most significant results of this paper seem to be that one can now determine the mass of a single star. Using pure Newtonian mechanics, one can only make estimates of the mass of one member of a binary. It is through the mechanical and electromagnetic coupling which makes the determination possible.

Of course one could also follow the scaling hypothesis to use $\alpha L$ to replace $M$ in equation (4). Then we have

$$\frac{4}{3} \frac{d\omega}{df} = \frac{16\alpha^2}{3c^2} I,$$

where $I$ is the moment of inertia of the pulsar.

A plot of the quantity $(4/\omega^3)(-d\omega/df)$ versus $\omega$ for pulsars yields the distribution of their moments of inertia. It turns out that the data points for the 202 known pulsars spread over six orders of magnitude, which does not seem to comply with any one of the currently suggested neutron star models, implying incompatibility between equations (2) and (4). The basic difference between equations (10) and (3) is that in equation (10) the observed quantities are related to the values of $I$ instead of $I/R_0$ as in equation (3). The spread of points in Figure 3 is mostly due to the fact that $R_0$ increases with the decrease of pulsed mass.

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