NONSTEADY INTERACTION OF PLASMA WITH BODIES MOVING IN SPACE

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Abstract. Nonsteady interactions between spacecraft and plasma are investigated in detail. The system of equations describing these interactions is obtained. It is shown that an electromagnetic soliton is excited via the modulational instabilities, which result from the radiation of antenna systems on the body which are the source of waves. In the meantime the density in the far wake diminishes, and its disturbance becomes also a soliton if the pump wave is sufficiently intense.

1. Introduction

The object of this investigation has been to study the interaction that occurs when a conducting body moves through ionosphere at mesothermal speeds. Since the launching of the satellites into near-Earth and outer space, great interest has been aroused in the study of the interaction effects. Many articles and monographs have been devoted to theoretical studies of the effects in the vicinity of a body moving through the ionosphere (Al’pert et al., 1965; Gurevich et al., 1969; Liu, 1969; Al’pert, 1983). However, as indicated by Al’pert (1983), a serious shortcoming of the present state of the theory is the absence of any studies of nonsteady-state problems.

Plasma waves and instabilities excited in the vicinity of a body moving in ionosphere represent typically nonsteady-state problems. Indeed, a large-amplitude solitary wave, which had excited in the ionosphere when the Apollo spacecraft was launched, was observed (Bakai et al., 1977). Therefore, studies of the nonsteady-state problems is very important.

The characteristic dimensions of bodies moving in the upper ionosphere (for example, rocket, satellite, and missile) are of the order of one or a few meters. The mean-free-path $L$ of the particles here is considerably larger ($L > 10^4$ cm). In other words, a dominant characteristic of the dynamics is its large Knudsen number, i.e., $L/R_0 \gg 1$. Therefore, it is necessary to use kinetic theory in order to describe the processes in the vicinity of moving bodies.

On the other hand, a characteristic of the bodies moving in the ionosphere is its mesothermal speed which implies that $v_{Ti} \ll v_0 \ll v_{Te}$ ($v_{Te}$ and $v_{Ti}$ are thermal velocities of electrons and ions). Under this condition the body only very weakly disturbs the equilibrium of electrons: i.e., the kinetic effects for electrons are negligible. Hence, we can use the hydrodynamical equations for electron description, and the ion distribution function $f_i$ should obey Vlasov’s equation. In addition, the quasi-neutral approximation is valid because the directed kinetic energy ($E = \frac{1}{2}m_i v_i^2$) is much larger than the electrostatic potential energy due to electric field effect; and in many of the ionospheric
dynamic problems of interest, the geomagnetic field effect is almost negligible (Liu, 1969). In Sections 2 and 3 we will study the coupling between electron motion and the field with high-frequency. The kinetic effects for ions and nonlinear coupling effects for the field and disturbed density are examined in Section 4. Several problems, including modulational instability, are discussed in Section 5.

2. Effects of High-Frequency Field on the Motion of Electrons

At small distances from a radiating antenna (radiation frequency $\omega > \omega_{pe}$), the intensity of the field with high-frequency may get extremely high. This leads to the effect of the field on electron motion.

The motion of electrons is governed by the equations

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0,$$  \hspace{1cm} (2.1)

$$\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \nabla) \mathbf{v}_e = \frac{e}{m_e} \left( \mathbf{E} + \frac{1}{c} \mathbf{v}_e \times \mathbf{B} \right) - \frac{1}{m_e} \frac{\gamma_e T_e}{n_e} \nabla n_e,$$  \hspace{1cm} (2.2)

$$\nabla \times \mathbf{E} = - \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},$$  \hspace{1cm} (2.3)

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} (en_e \mathbf{v}_e - en_i \mathbf{v}_i),$$  \hspace{1cm} (2.4)

$$\nabla \cdot \mathbf{B} = 0;$$  \hspace{1cm} (2.5)

all above symbols have usual meanings.

On the basis of the two time-scale approximations (Li, 1985), all the field quantities could be distinguished as the fast time-scale ($\sim \omega_{pe}^{-1}$) and slow time-scale ($\sim \omega_{pi}^{-1}$) components

$$\mathbf{A} = (n_e, \mathbf{v}_e, P_e, \mathbf{E}, \mathbf{B}) = \mathbf{A}_f + \mathbf{A}_s,$$  \hspace{1cm} (2.6)

and it would be assumed that the assembly average value of fast time-scale components over the slow time-scale vanish: i.e.,

$$\langle A_f \rangle = 0.$$  \hspace{1cm} (2.7)

One can deduce from Equation (2.1) that

$$\frac{\partial}{\partial t} (n_s + n_f) + \nabla \cdot [(n_s + n_f) (\mathbf{v}_s + \mathbf{v}_f)] = 0.$$  \hspace{1cm} (2.8)

The assembly average of Equation (2.8) over the slow time-scale becomes

$$\frac{\partial}{\partial t} n_s + \nabla \cdot [n_s \mathbf{v}_s + \langle n_f \mathbf{v}_f \rangle] = 0.$$  \hspace{1cm} (2.9)
Equation (2.9) subtracted from Equation (2.8) becomes

\[
\frac{\partial}{\partial t} n_f + \nabla \cdot (n_s v_f + n_f v_f + n_f v_s - \langle n_f v_f \rangle) = 0 .
\]  

(2.10)

From Equation (2.2), we obtain the lowest-order component equation for the fast time-scale, as shown by

\[
\frac{\partial}{\partial t} v_f = \frac{1}{m_e} e E_f .
\]  

(2.11)

By use of Equation (2.11), one can estimate the terms in Equation (2.10) as

\[
\left| \frac{\nabla \cdot (n_f v_f)}{\partial n_f / \partial t} \right| \sim \frac{k n_f v_f}{\omega n_f} \sim \frac{k}{\omega m_e \omega} \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \left( \frac{E_f}{W^{1/2}} \right) ,
\]  

(2.12a)

\[
\left| \frac{\nabla \cdot (n_f v_s)}{\partial n_f / \partial t} \right| \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right) \left( \frac{v_s}{v_{Te}} \right) ,
\]  

(2.12b)

where \( k_d \) is the Debye wave-number and

\[
\bar{W} = E_f^2 / (4 \pi n_e^0 T_e) ;
\]  

(2.12c)

for the case of interest to us, we should expect that

\[
\bar{W} \ll 1 .
\]  

(2.12d)

For the slow time-scale fluid motion we have

\[
\left| v_s \right| \ll v_{Te} .
\]  

(2.13)

Hence, in the case of

\[
v_{Te} / n_\phi \ll 1 , \quad (n_\phi \equiv \omega / k)
\]  

(2.14a)

and

\[
v_{Te} / n_\phi \ll \left( \frac{\omega}{\omega_{pe}} \right) \bar{W}^{-1/2} ,
\]  

(2.14b)

Equation (2.10) could be simplified as

\[
\frac{\partial n_f}{\partial t} + \nabla \cdot (n_s v_f) = 0 .
\]  

(2.15)

By combining Equations (2.11) and (2.15), we obtain the estimate values

\[
v_f \sim \frac{|e| E_f}{m_e \omega} \sim \frac{\omega_{pe}}{\omega} \bar{W}^{1/2} ,
\]  

(2.16a)
\[
\frac{n_f/n_s}{n_f v_f} \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \left( \frac{\omega_{pe}}{\omega} \right) \left( \frac{v_{Te}}{v_s} \right) \ll 1. \tag{2.16b}
\]

By use of (2.13) and (2.16), one can estimate the terms in Equation (2.9) from
\[
\left| \frac{n_f v_f}{n_s v_s} \right| \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \left( \frac{\omega_{pe}}{\omega} \right) \left( \frac{v_{Te}}{v_s} \right) \sim \left( \frac{v_{Te}}{v_\phi} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \left( \frac{v_{Te}}{v_s} \right) \ll 1,
\]
thus Equation (2.9) becomes
\[
\frac{\partial}{\partial t} n_s + \nabla \cdot (n_s v_s) = 0. \tag{2.18}
\]

Now, we study the momentum equation for the electron. Substituting the corresponding fast and slow components into Equation (2.2), one obtains the average equation
\[
\frac{\partial}{\partial t} v_s + (v_s \nabla) v_s + \langle (v_f \nabla) v_f \rangle =
\]
\[
= \frac{e}{m_e} \left[ E_s + \frac{1}{c} v_s \times B_s + \langle v_f \times \frac{1}{c} B_f \rangle \right] - \frac{\gamma_e T_e v_{Te} n_e}{m_e n_s},
\tag{2.19}
\]
and the fast-component equation
\[
\frac{\partial}{\partial t} v_f + (v_s \nabla) v_f + (v_f \nabla) v_s + [(v_f \nabla) v_f - \langle (v_f \nabla) v_f \rangle] + \frac{\gamma_e T_e v_{Te} n_f}{m_e n_s} =
\]
\[
= \frac{e}{m_e} \left[ E_f + \frac{1}{c} v_s \times B_f + \frac{1}{c} v_f \times B_s + \frac{1}{c} v_f \times B_f - \langle \frac{1}{c} v_f \times B_f \rangle \right].
\tag{2.20}
\]
If we compare the terms in Equation (2.20) we find that
\[
\left| \frac{(v_s \nabla) v_f}{\partial v_f/\partial t} \right| \sim \frac{k}{\omega} v_s - \left( \frac{v_{Te}}{v_\phi} \right) \left( \frac{v_f}{v_{Te}} \right) \ll 1, \tag{2.21a}
\]
\[
\left| \frac{(v_s \nabla) v_s}{\partial v_f/\partial t} \right| \sim \frac{k_s}{\omega} v_s \sim \left( \frac{v_{Te}}{v_\phi} \right) \left( \frac{v_s}{v_{Te}} \right) \left( \frac{k_s}{k} \right), \tag{2.21b}
\]
\[
\left| \frac{(v_f \nabla) v_f}{\partial v_f/\partial t} \right| \sim \left( \frac{v_{Te}}{v_\phi} \right) \left( \frac{v_f}{v_{Te}} \right) \ll 1, \tag{2.21c}
\]
\[
\left| \frac{B_f - v_{f,s}}{\partial E_f} \right| \sim \left( \frac{v_{Te}}{v_\phi} \right) \left( \frac{v_{f,s}}{v_{Te}} \right) \ll 1, \tag{2.21d}
\]
\[
\left| \frac{\left( e/m_e c \right) \left( \mathbf{B}_s \times \mathbf{v}_f \right)}{\partial \mathbf{v}_f / \partial t} \right| \sim \left( \begin{array}{c} \omega_{Be} \\ \omega_{pe} \\ \omega \end{array} \right) \left( \begin{array}{c} \omega_{pe} \\ \omega \end{array} \right).
\]

(2.21e)

One can assume that the characteristic scale of slow component is larger than one of fast component – i.e.,

\[ k_s < k. \]

Thus (2.21b) becomes

\[
\left| \frac{\left( \mathbf{v}_f \cdot \nabla \right) \mathbf{v}_f}{\partial \mathbf{v}_f / \partial t} \right| \ll 1,
\]

so that Equation (2.20) reduces to

\[
\frac{\partial}{\partial t} \mathbf{v}_f = \frac{e}{m_e} \mathbf{E}_f - \frac{\gamma_e T_e}{m_e n_s} \nabla n_f + \frac{e}{m_e c} \mathbf{v}_f \times \mathbf{B}_s.
\]

(2.22)

In the ionosphere, the geomagnetic field is very weak and one has generally

\[ \omega_{pe} \gg \omega_{Be}. \]

In this case, as seen from (2.21e), one can ignore the last term on Equation (2.22) – i.e.,

\[
\frac{\partial}{\partial t} \mathbf{v}_f = \frac{e}{m_e} \mathbf{E}_f - \frac{\gamma_e T_e}{m_e n_s} \nabla n_f.
\]

(2.24)

It follows from (2.3) and (2.22) that

\[
\mathbf{B}_f = -\frac{m_e c}{e} \nabla \times \mathbf{v}_f + \nabla \times (\phi_e \times \mathbf{B}_s)
\]

(2.25)

and

\[
\mathbf{v}_f = \frac{\partial}{\partial t} \phi_e.
\]

(2.26)

If we note that

\[
\left( \mathbf{v}_f \cdot \nabla \right) \mathbf{v}_f = \frac{1}{2} \nabla (\mathbf{v}_f)^2 - \mathbf{v}_f \times (\nabla \times \mathbf{v}_f)
\]

and use (2.25), Equation (2.19) is transformed into

\[
\frac{\partial}{\partial t} \mathbf{v}_s + \left( \mathbf{v}_s \cdot \nabla \right) \mathbf{v}_s = \frac{e}{m_e} \left[ \mathbf{E}_s + \frac{1}{c} \mathbf{v}_s \times \mathbf{B}_s \right] - \frac{\gamma_e T_e}{m_e n_s} \nabla n_s + \mathbf{F}_p,
\]

(2.27)

where \( \mathbf{F}_p \) is ponderomotive force due to the high-frequency plasmons and MHD interaction – i.e.,

\[
\mathbf{F}_p = -\frac{1}{2} \nabla \left( \left( \mathbf{v}_f \right)^2 \right) + \frac{e}{m_e c} \left( \mathbf{v}_f \times \nabla \times (\phi_e \times \mathbf{B}_s) \right).
\]

(2.28)
For the case of interest to us, the geomagnetic field effect is almost negligible, then (2.27) becomes

\[ \frac{\partial}{\partial t} v_s + (v_s \nabla) v_s = \frac{e}{m_e} E_s - \frac{\gamma_e T_e}{m_e n_s} \nabla n_s - \frac{1}{2} \nabla \left( \langle v_f^2 \rangle \right). \]  

(2.29)

3. The Transport Equation for Fast Oscillations

One may ignore the fast component of ions in Equation (2.4) due to its large inertia and retain

\[ \nabla \times B_f = \frac{1}{c} \frac{\partial E_f}{\partial t} + \frac{4\pi e}{c} \left[ n_s v_f + n_f v_f + n_f v_s - \langle n_f v_f \rangle \right]; \]  

(3.1)

from (2.13) and (2.16) it can also be shown that

\[ \left| \frac{n_f v_s}{n_s v_f} \right| \sim \frac{n_f}{n_s} \ll 1; \]

then (3.1) becomes

\[ \nabla \times B_f = \frac{1}{c} \frac{\partial}{\partial t} E_f + \frac{4\pi e}{c} n_s v_f; \]  

(3.2)

from Maxwell's equation

\[ \nabla \cdot E = 4\pi (e n_e - e n_i) \]

one gets

\[ \nabla \cdot E_f = 4\pi e n_f, \]  

(3.3)

so that Equation (2.24) becomes

\[ \frac{\partial}{\partial t} v_f = \frac{e}{m_e} E_f - \frac{\gamma_e T_e}{4\pi n_s m_e} \nabla (\nabla \cdot E_f). \]  

(3.4)

By combining Equations (2.3), (3.2), and (3.4), we arrive at the equation

\[ \frac{\partial^2}{\partial t^2} E_f + c^2 \nabla \times \nabla \times E_f + \frac{4\pi e^2}{m_e} n_s E_f - \gamma_e v_{Te}^2 \nabla (\nabla \cdot E_f) = 0, \]  

(3.5)

where we considered that time-change of the term \( n_s v_f \) mostly depends on the change of fast time-scale – i.e.,

\[ \frac{\partial}{\partial t} (n_s v_f) = n_s \frac{\partial}{\partial t} v_f. \]
If we set
\[ n_s = n_0 + \delta n, \quad (3.6) \]

Equation (3.5) becomes
\[ \frac{\partial^2}{\partial t^2} E_f + c^2 \nabla \times \nabla \times E_f + \omega_{pe}^2 E_f + \frac{\delta n}{n_0} \omega_{pe}^2 E_f - \gamma_e v_{Te}^2 \nabla (\nabla \cdot E_f) = 0, \quad (3.7) \]

where
\[ \omega_{pe}^2 = 4\pi e^2 n_0/m_e \quad (3.8) \]

is the Langmuir frequency in undisturbed state. For longitudinal waves, linearization of Equation (3.7) gives the dispersion relation
\[ \omega^2 = \omega_{pe}^2 + \gamma_e k^2 v_{Te}^2. \]

Comparing it with the dispersion relation of the Langmuir wave, one can take \( \gamma_e = 3 \).

If we write
\[ v_f = \frac{1}{2} [v(r, t) e^{i\omega t} + v^*(r, t) e^{-i\omega t}], \quad (3.9a) \]
\[ E_f = \frac{1}{2} [E(r, t) e^{i\omega t} + E^*(r, t) e^{-i\omega t}], \quad (3.9b) \]

where the amplitudes \( v(r, t) \) and \( E(r, t) \) are slowly varying functions over time; and using (2.11) one easily obtains
\[ \langle (v_f)^2 \rangle = \frac{e^2}{2m_e^2 \omega^2} |E(r, t)|^2. \quad (3.10) \]

Now by use of (3.9b) with
\[ \frac{1}{\omega_{pe}} \left| \frac{\partial}{\partial t} \ln E(r, t) \right| \ll 1, \quad (3.11) \]

we find from (3.7) that the amplitude transport equation of fast-varying field for \( \omega \approx \omega_{pe} \) becomes
\[ 2i\omega_{pe} \frac{\partial}{\partial t} E_c + c^2 \nabla \times \nabla \times E_c - 3v_{Te}^2 \nabla (\nabla \cdot E_c) + \frac{\delta n}{n_0} \omega_{pe}^2 E_c = 0. \quad (3.12) \]

Substituting (3.10) into (2.29) and using (3.6) we obtain
\[ \frac{\partial}{\partial t} v_s + (v_s \nabla) v_s = -\frac{e}{m_e} \nabla \phi - \frac{\gamma_e T_e}{m_e} \nabla (n_0 + \delta n) - \frac{1}{16\pi n_0 m_e} \nabla (|E|^2), \quad (3.13) \]
where $\phi$ is electrostatic potential

$$E_s = -\nabla \phi.$$  \hfill (3.14)

For small disturbances,

$$|\delta n| \ll n_0, \quad (n_0 = \text{const.}).$$  \hfill (3.15)

Linearization of Equations (3.13) and (3.18) yields

$$\left( \frac{\partial^2}{\partial t^2} - \gamma_e v_e^2 \nabla^2 \right) \frac{\delta n}{n_0} = \frac{e}{m_e} \nabla^2 \phi + \frac{1}{m_e} \nabla^2 \left( \frac{|E|^2}{16\pi n_0} \right).$$  \hfill (3.16)

For the static limit, one may ignore the first term on the left-hand side of Equation (3.16) to obtain

$$\frac{\delta n}{n_0} = -\frac{U_{\text{eff}} + e\phi}{\gamma_e T_e},$$  \hfill (3.17)

where

$$U_{\text{eff}} = \frac{1}{16\pi n_0} |E|^2$$  \hfill (3.18)

is the potential resulting from high-frequency field.

In fact (3.17) gives the Boltzmann distribution of the electrons in the field composed of ponderomotive force and electrostatic force.

4. Far-Wake Ions Flow

To obtain Equations (3.12), (3.17), and (3.18), we have to study the influence of the slowly-varying field $\phi$ on ions flow. In addition, since the body’s velocity is considerably greater than the thermal velocity of the ion, a disturbance of the ions occurs basically because of their interaction with the body itself. In a coordinate system moving with the body, the collisionless Boltzmann equation for ionic distribution is of the form

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{r}} - \frac{\partial f_i}{\partial \mathbf{P}} \cdot \frac{\partial}{\partial \mathbf{r}} \left( e_i \phi + U \right) = 0,$$  \hfill (4.1)

where $U$ is an energy of interaction between ions and the body.

Let us assume that

$$f_i = f_0 + \delta f,$$

where $f_0$ is a balance distribution of the free-stream, the normalization of which is

$$n_0 = \int f_0 \frac{d\mathbf{P}}{(2\pi)^3},$$  \hfill (4.2)

and the perturbation $\delta f$ is small compared with $f_0$. 

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Substituting (4.2) into (4.1) we obtain
\[
\frac{\partial}{\partial t} \delta f + v \frac{\partial}{\partial r} \delta f - \frac{\partial f_0}{\partial P} \frac{\partial}{\partial r} (e_i \phi + U) = \frac{\partial(\delta f)}{\partial P} \frac{\partial}{\partial r} (e_i \phi + U). \tag{4.3}
\]

By changing over to the Fourier component
\[
A(r, t) = A_{\Omega, K} e^{-i\Omega t - iK \cdot r} dK d\Omega \equiv \int A_K e^{-i\Omega t - iK \cdot r} dK \tag{4.4}
\]
and
\[
A_K = \int A(r, t) e^{i\Omega t - iK \cdot r} dr dt/(2\pi)^4, \tag{4.5}
\]
as well as to a fixed coordinate system
\[
u = v + V_0, \quad P_u = m_i u, \tag{4.6}
\]
where \(V_0\) is the velocity of the body, one obtains from (4.3) that
\[
-i[\Omega - K(u - V_0)](\delta f)_K = i(e_i \phi_K + U_K)K \frac{\partial f_0}{\partial P_u} + \]
\[
\left[ \frac{\partial(\delta f)}{\partial P_u} \frac{\partial}{\partial r} (e_i \phi + U) \right]_K. \tag{4.7}
\]

For the far-wake which corresponds to small values of \(K\), one can ignore the second term on the right-hand side of Equation (4.7) because the energy \(U_K\) is the limit for small values of \(K\) (Pitayevskii and Kresin, 1961). Therefore, for the far-wake Equation (4.7) becomes
\[
(\delta f)_K = -K \frac{\partial f_0}{\partial P_u} e_i \phi_K \frac{I_{\Omega}(P_u)}{(\Omega + K \cdot V_0 - K \cdot u)} + i \frac{I_{\Omega}(P_u)}{\Omega + K \cdot V_0 - K \cdot u}, \tag{4.8}
\]
where
\[
I_{\Omega}(P_u) = \lim_{K \to 0} I_{\Omega, K}(P_u) = \lim_{K \to 0} \left[ \frac{\partial(\delta f)}{\partial P_u} \frac{\partial}{\partial r} (e_i \phi + U) \right]_K. \tag{4.9}
\]

It follows from (4.8) that
\[
\frac{(\delta n_i)_K}{n_0} = \frac{1}{n_0} \int (\delta f)_K \frac{dP_u}{(2\pi)^3} = \]
\[
= -e_i \phi_K \int \frac{K \cdot (\delta f_0/\partial P_u)}{\Omega_0 - K \cdot u + i\epsilon} \frac{dP_u}{(2\pi)^3} + i \int \frac{I_{\Omega}(P_u)}{\Omega_0 - K \cdot u + i\epsilon} \frac{dP_u}{(2\pi)^3}, \tag{4.10}
\]
where

$$\Omega_0 = \Omega + \mathbf{K} \cdot \mathbf{V}_0$$  \hspace{1cm} (4.11)

and the term \(i\varepsilon\) in denominators of integrated functions arises from the Landau rule (e.g., Li, 1987). By use of (3.17), the quasi-neutrality condition has the form

$$\frac{(\delta n)_K}{n_0} = \frac{\varepsilon}{n_0} \left[ - \frac{L}{\gamma_e T_e} (U_{\text{eff}})_K + e \phi_K \right].$$  \hspace{1cm} (4.12)

By combining Equations (4.10) and (4.12), we obtain

$$\frac{(\delta n)_K}{n_0} = \frac{1}{1 + L} \left[ - \frac{L}{\gamma_e T_e} (U_{\text{eff}})_K + i Q_K \right],$$  \hspace{1cm} (4.13)

where

$$L = \frac{\gamma_e T_e}{n_0} \int \frac{K \cdot (\partial f_0/\partial \mathbf{P}_u)}{\Omega_0 - \mathbf{K} \cdot \mathbf{u} + i\varepsilon \,(2\pi)^3} \, d\mathbf{P}_u$$  \hspace{1cm} (4.14)

and

$$Q_K = \frac{1}{n_0} \int \frac{I_{\Omega}(\mathbf{P}_u)}{\Omega_0 - \mathbf{K} \cdot \mathbf{u} + i\varepsilon \,(2\pi)^3} \, d\mathbf{P}_u.$$  \hspace{1cm} (4.15)

The dielectric constant of plasma is defined by

$$\varepsilon_K^{\varepsilon} = 1 + \frac{4\pi e^2}{K^2} \int \frac{K \cdot (\partial f_0/\partial \mathbf{P}_u)}{\Omega_0 - \mathbf{K} \cdot \mathbf{u} + i\varepsilon \,(2\pi)^3} \, d\mathbf{P}_u;$$  \hspace{1cm} (4.16)

and in terms of it, Equation (4.14) becomes

$$L = \frac{\gamma_e T_e}{n_0} \frac{K^2}{4\pi e^2} \left( \varepsilon_K^{\varepsilon} - 1 \right).$$  \hspace{1cm} (4.17)

If the distribution of the free stream is Maxwellian distribution

$$f_0 = n_0 \frac{(2\pi)^{3/2}}{(m_1 V_{Ti})^3} \exp \left[ - \frac{\mathbf{P}_u^2}{2m_1 V_{Ti}^2} \right],$$  \hspace{1cm} (4.18)

then one has (e.g., Li, 1987)

$$\varepsilon_K^{\varepsilon} = 1 + \frac{\alpha_{pi}^2}{K^2 V_{Ti}^2} \left[ 1 - Z \left( \frac{\Omega_0}{\sqrt{2} K V_{Ti}} \right) \right],$$  \hspace{1cm} (4.19)
where the $Z$-function is defined

\[
Z(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{x}{x - \xi + i\epsilon} e^{-\xi^2} d\xi.
\]  

(4.20)

For this case of low frequency - i.e.,

\[
V_{Ti} \gg \frac{\Omega_0}{K},
\]  

(4.21a)

\[
\Omega_0 = \Omega + K \cdot V_0 \approx K \cdot V_0,
\]  

(4.21b)

using the approximate expression

\[
Z(x) = 2x^2 - i \sqrt{\pi} x \quad (|x| \ll 1),
\]  

(4.22)

we obtain

\[
L \approx \frac{\gamma_e T_e}{T_i};
\]  

(4.23)

Equation (4.13) then becomes

\[
\frac{(\delta n)_K}{n_0} = \frac{-(U_{\text{eff}})_K}{\gamma_e T_e + T_i} + i \frac{T_i}{\gamma_e T_e + T_i} Q_K.
\]  

(4.24)

In the time-space representation Equation (4.24) becomes

\[
\frac{\delta n}{n_0} = -\frac{1}{\gamma_e T_e + T_i} U_{\text{eff}} + \frac{T_i}{\gamma_e T_e + T_i} Q,
\]  

(4.25)

where

\[
Q = i \int Q_K e^{-i\Omega t + iK \cdot r} d\Omega dK.
\]  

(4.26)

By use of (4.15) and (4.12), Equation (4.26) is led up to

\[
Q = \frac{i}{n_0} \int dK e^{iK \cdot r} \int \frac{dP_u}{(2\pi)^3} \frac{1}{\Omega_0 - K \cdot u + i\epsilon} \int I_{\Omega}(P_u) e^{-i\Omega t} d\Omega.
\]  

(4.27)

Taking note of (4.8) and using Fourier inverse transform in the $K$-space

\[
\mathcal{L}_K = \int \mathcal{L}(r, t) e^{-iK \cdot r} \frac{dr}{(2\pi)^3},
\]  

(4.28)

one has

\[
\int I_{\Omega}(P_u) e^{-i\Omega t} d\Omega = \lim_{K \to 0} \left[ \frac{\partial}{\partial P_u} \frac{\partial}{\partial r} (e_i \phi + U) \right]_K =
\]
\[
\int \frac{\partial (\delta f)}{\partial P_u} \frac{\partial}{\partial r} (\epsilon_j \phi + U) \frac{d\mathbf{r}}{(2\pi)^3}. 
\] (4.29)

The expression on the right-hand side of above last equation is just proportional to \(I\)-function defined by Pitayevskii and Kresin (1961). If we take (cf. Al'pert et al., 1965)

\[
I = (2\pi)^3 \int I_\Omega(P_u) e^{-i\alpha t} d\Omega = -f_0(P_u) V_0 S_{\text{eff}}, 
\] (4.30)

where \(S_{\text{eff}}\) is the effective cross-section of the body for scattering of particles by the body and by the field \(\phi\), and

\[
S_{\text{eff}} \approx \pi R_0^2, 
\] (4.31)

then

\[
Q = -\frac{\pi R_0^2}{n_0} V_0 i \int \frac{dK}{(2\pi)^3} e^{i \mathbf{K} \cdot \mathbf{r}} \frac{n_0}{\Omega_0} Z \left( \frac{\Omega_0}{\sqrt{2 K V_{T_i}}} \right). 
\] (4.32)

By writing the relation between the \(Z\)-function and probability function

\[
\frac{iZ(x)}{x} = e^{-x^2} \left( \sqrt{\pi} + 2i \int_0^x e^{t^2} dt \right) \equiv \eta(x), 
\] (4.33)

and by use of (4.21b), Equation (4.32) becomes

\[
Q = -\frac{\pi R_0^2 V_0}{\sqrt{2} V_{T_i}} \int \frac{dK}{(2\pi)^3} e^{i \mathbf{K} \cdot \mathbf{r}} \frac{\eta \left( \frac{\mathbf{K} \cdot \mathbf{V}_0}{\sqrt{2 K V_{T_i}}} \right)}{K}. 
\] (4.34)

Taking (4.28) into account, we have

\[
\int Q e^{-i \mathbf{K} \cdot \mathbf{r}} \frac{d\mathbf{r}}{(2\pi)^3} = -\frac{V_0}{\sqrt{2} V_{T_i}} \frac{\pi R_0^2}{(2\pi)^3} \frac{\eta \left( \frac{\mathbf{K} \cdot \mathbf{V}_0}{\sqrt{2 K V_{T_i}}} \right)}{K}. 
\] (4.35)

For far-wake (\(\mathbf{Z} \parallel \mathbf{V}_0\))

\[
K_x^2 + K_y^2 \gg K_z^2, 
\] (4.36)

Equation (4.35) becomes

\[
\int_{z < 0} Q e^{-i \mathbf{K} \cdot \mathbf{r}} d\mathbf{r} = -\frac{\pi R_0^2}{\sqrt{2} V_{T_i}} V_0 \frac{\eta \left( \frac{K_x V_0}{\sqrt{2 K_x^2 + K_y^2}} \right)}{\sqrt{K_x^2 + K_y^2}}; 
\] (4.37)
then one finds easily that
\[
Q = - \frac{V_0^2}{V_{Ti}^2} \frac{\pi R_0^2}{2\pi z^2} \exp \left( - \frac{V_0^2}{2V_{Ti}^2} \frac{x^2 + y^2}{z^2} \right).
\]  
(4.38)

Therefore, substituting (3.18) and (4.38) into Equation (4.25) we arrive at
\[
\frac{\partial n}{n_0} = - \frac{|E|^2}{16n_0(\gamma_e T_e + T_i)} - \frac{T_i}{\gamma_e T_e + T_i} \left( \frac{V_0}{V_{Ti}} \right)^2 \frac{\pi R_0^2}{2\pi z} \times
\exp \left( - \frac{V_0^2}{2V_{Ti}^2} \frac{x^2 + y^2}{z^2} \right),
\]  
(4.39)

In terms of the dimensionless variables
\[
\tilde{f} = \frac{2}{3} \frac{\omega_{pe}}{C_s} \mu r, \quad \tau = \frac{2}{3} \frac{\mu}{\omega_{pe}} t, \quad n = \frac{3}{4\mu} \frac{\partial n}{n_0},
\]  
(4.40a)

\[
\tilde{E} = \frac{\sqrt{3} E}{8[\pi n_0 \mu (\gamma_e T_e + T_i)]^{1/2}}, \quad \tilde{V}_0 = \frac{V_0}{C_s}
\]  

and
\[
\mu = m_e/m_i, \quad \alpha = C_s^2/3v_{Te}^2, \quad C_s^2 = T_e/m_i,
\]  
(4.40b)

Equations (3.12) and (4.39) may be written as
\[
i \frac{\partial}{\partial \tau} \tilde{E} + \alpha \nabla \times \nabla \times \tilde{E} - \nabla(\nabla \cdot \tilde{E}) + n\tilde{E} = 0,
\]  
(4.41)

\[
n = - |\tilde{E}|^2 - \frac{3}{4\mu} \frac{T_e}{\gamma_e T_e + T_i} \frac{\pi R_0^2}{2\pi z^2} \exp \left( - \frac{T_e}{2T_i} \frac{\tilde{v}_0^2}{\tilde{z}^2} \frac{x^2 + y^2}{z^2} \right).
\]  
(4.42)

Equations (4.41) and (4.42) describe the nonlinear coupling of interest to us between high-frequency field and density disturbance.

**5. Modulation Instability**

For the far-wake and strong radiation source (for example, radiating antenna), as the first approximation we may ignore the second term on the right-hand side of Equation (4.42):
\[
n^{(1)} = - |\tilde{E}|^2,
\]  
(5.1)

where \( \tilde{E} \) satisfies the equation
\[
i \frac{\partial}{\partial \tau} \tilde{E} + \alpha \nabla \times \nabla \times \tilde{E} - \nabla(\nabla \cdot \tilde{E}) + n^{(1)}\tilde{E} = 0.
\]  
(5.2)
The next approximation for the density disturbance is

\[ n = n^{(1)} - \frac{3}{4 \mu} \frac{T_e}{\gamma_e T_e + T_i} \delta_0^2 \frac{\pi R_0^2}{2 \pi e^2} \exp \left( \frac{-T_e}{2T_i} \delta_0^2 \frac{\delta^2 + \gamma^2}{\varepsilon^2} \right). \]  

(5.3)

Now we study the instability of a finite amplitude transverse wave for this case of the first approximation. By rewriting Equations (5.1) and (5.2) as

\[ -\nabla^2 n^{(1)} = \nabla^2 \left| \mathbf{\hat{E}} \right|^2, \]  

(5.4)

\[ i \mathbf{\hat{E}}_\perp - \alpha \nabla \times \nabla \times \mathbf{\hat{E}} + \nabla (\nabla \cdot \mathbf{\hat{E}}) - n^{(1)} \mathbf{\hat{E}} = 0, \]  

(5.5)

where we used the complex conjugate of Equation (5.2). It is easily seen that the exact solutions of Equations (5.4) and (5.5) are

\[ \mathbf{\hat{E}}_1 = \mathbf{E}_0 \exp \left[i(\mathbf{k}_0 \cdot \mathbf{r} - \omega_0 \tau)\right], \quad n_1 = 0, \]  

(5.6a)

\[ \omega_0 = \alpha k_0^2, \quad \mathbf{k}_0 \cdot \mathbf{E}_0 = 0, \]  

(5.6b)

where \( \mathbf{E}_0 \) is the amplitude of pump wave resulting from the radiating antenna on the body. Linearizing Equations (5.4) and (5.5) we obtain

\[ -\nabla^2 n_{1\perp} = \nabla^2 \left( \mathbf{\hat{E}}_1 (\delta \mathbf{\hat{E}})^* + \mathbf{\hat{E}}_1^\perp (\delta \mathbf{\hat{E}}) \right), \]  

(5.7a)

\[ i (\delta \mathbf{\hat{E}})_\perp + \nabla (\nabla \cdot \delta \mathbf{\hat{E}}) - \alpha [\nabla \times \nabla \times \delta \mathbf{\hat{E}}] - n_{1\perp} \mathbf{\hat{E}}_1 = 0. \]  

(5.7b)

We shall study the perturbation states

\[ \delta \mathbf{\hat{E}} = (E_1 e_1 + E_2 e_2) \exp \left[i(k_+ \cdot \mathbf{\hat{r}} - \omega_+ \tau)\right] + \]  

\[ + (E_1^* e_1^+ + E_2^* e_2^+) \exp \left[-i(k_- \cdot \mathbf{\hat{r}} - \omega_- \tau)\right], \]  

(5.8a)

\[ n_{1\perp} = \frac{1}{2} n \left\{ \exp \left[i(\mathbf{k} \cdot \mathbf{r} - \omega \tau)\right] + \exp \left[-i(\mathbf{k} \cdot \mathbf{r} - \omega \tau)\right] \right\}, \]  

(5.8b)

where

\[ e_1 \parallel k_+, \quad e_1^+ \parallel k_-; \quad e_2 \perp k_+, \quad e_2^+ \perp k_-, \]  

(5.9a)

\[ k_\pm = k \pm k_0, \quad \omega_\pm = \omega \pm \omega_0. \]  

(5.9b)

Substituting (5.8) and (5.6) into (5.7b) and into its conjugate equation yields

\[ (\omega_+ - k_+^2) E_1 e_1 + (\omega_+ - \alpha k_+^2) E_2 e_2 = \frac{n}{2} \mathbf{E}_0, \]  

(5.10a)

\[ -(\omega_- + k_-^2) E_1 e_1^+ - (\omega_- + \alpha k_-^2) E_2 e_2^+ = \frac{n}{2} \mathbf{E}_0^*; \]  

(5.10b)

similarly one gets from (5.7a)

\[ -\frac{n}{2} = E_2 (\mathbf{E}_0 \cdot e_2^+ ) + E_1 (\mathbf{E}_0^* \cdot e_1^+ ) + E_2 (\mathbf{E}_0^* \cdot e_2) + E_1 (\mathbf{E}_0 \cdot e_1). \]  

(5.11)
From (5.10) and (5.11) we find that

$$1 = |E_0|^2 \left[ \frac{\cos^2 \theta_+}{-\omega - \alpha k_0^2 + (k_0 + k)^2} + \frac{\cos^2 \theta_-}{\omega - \alpha k_0^2 + (k - k_0)^2} + \frac{\sin^2 \theta_+}{-\omega - \alpha k_0^2 + \alpha(k + k_0)^2} + \frac{\sin^2 \theta_-}{\omega - \alpha k_0^2 + \alpha(k - k_0)^2} \right],$$  \hspace{1cm} (5.12)

where $\theta_{\pm}$ is the angle between $E_0$ and $(k \pm k_0)$. Now we study the case when $k$ is parallel to $k_0$ so that $\theta_{\pm} = \frac{1}{2} \pi$. This is the case of transverse perturbations across the field $E_0$. The dispersion relation then becomes

$$k^2 \left[ \omega - \alpha k_0^2 + \alpha(k + k_0)^2 \right] = k^2 |E_0|^2 \left[ \omega - \alpha k_0^2 + \alpha(k - k_0)^2 \right];$$  \hspace{1cm} (5.13)

when $|k| \gg |k_0|$, (5.13) reduces to

$$\omega^2 = \alpha^2 k^4 - 2 |E_0|^2 \alpha k^2.$$  \hspace{1cm} (5.14)

Similarly, the dispersion relation for longitudinal perturbation when $k \perp k_0$ and $|k| \gg \sqrt{\alpha} k_0$ is

$$\omega^2 = k^4 - 2 |E_0|^2 k^2.$$  \hspace{1cm} (5.15)

One can find the maximum growth rate $\gamma'_{\text{max}}$ from (5.15) to be

$$\gamma'_{\text{max}} = |E_0|^2, \quad (k'_{\text{max}} = |E_0|);$$  \hspace{1cm} (5.16)

while by using (5.14) the growth rate $\gamma'_\text{max}$ of transverse perturbation becomes

$$\gamma'_{\text{max}} = |E_0|^2, \quad (k'_{\text{max}} = \alpha^{-1/2} |E_0|).$$  \hspace{1cm} (5.17)

We note that the growth rate of transverse perturbations is the same as longitudinal one but the scale-length of the former is increased by a factor $\alpha^{1/2} \gg 1$.

Therefore, the transverse perturbations develop on a larger scalelength than the longitudinal one, and this will lead to pancake-like structures. It is well known that Equations (5.4) and (5.5) describe the nonlinear entity-caviton and the entity is a stable soliton in the one-dimensional case (see, e.g., Li, 1987):

$$\hat{E} = E_0 \text{ sech} \left[ \frac{\hat{z} - \hat{u} \tau}{\sqrt{2/E_0}} \right] e^{i \varphi},$$  \hspace{1cm} (5.18a)

$$\varphi = \left( \frac{E_0^2}{2} + \frac{\hat{u}^2}{4} \right) \tau + \frac{\hat{u}}{2} (\hat{z} - u \tau) + \varphi_0,$$  \hspace{1cm} (5.18b)

where $\hat{u}$ is the Mach number for the soliton. Owing to the fact that the ratio of the longitudinal to the transverse axis was about $\alpha^{-1/2} \sim 10^{-3}$, the thickness of this pancake-like structures is small. In this connection the term describing the variation of the
field in longitudinal direction plays the major role in Equation (5.5). Hence, it is tempting to assume that when the antenna on the body pumps energy into surrounding plasma the result is the formation of the caviton with longitudinally soliton shape (5.18).

6. Conclusions

From the above studies we arrive at the following conclusions:

(1) The interaction between a body in space and its highly rarefied plasma environment is one of the basic problems in space plasma physics; we obtain the system of Equations (4.41) and (4.42), describing the interaction for far-wake in a self-consistent.

(2) The motion of the body with antenna system in the ionosphere may directly excite electromagnetic soliton via the modulational instability. The result agree with those observed after the spacecraft has passed, quoted by Bakai et al. (1977).

(3) The soliton is a pancake-like structure with longitudinally a soliton shape, which is nearly stable and moves at a speed which is subsonic. Meanwhile the disturbance density in the far-wake represents also a kind of soliton of evacuation, if the radiation of the antenna as pump waves source is enough intense.

References