Atmospheric models of coronal hole network

Zhou Ai-hua, Fan Da-xiong, Lin Chun-mei, and Wang Jian-min

Purple Mountain Observatory, Academia Sinica

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Energy balance models of the chromosphere-corona transition region are computed for a segment of coronal hole assuming a nonradial magnetic field geometry. The energy fluxes considered include radiation, conduction, convection and mechanical wave (e.g., Alfvén waves) heating. The calculated results show that the temperature $T$ is about 60% lower, the electron density $N$ is twice smaller and the thickness of the transition region is four times larger in the coronal hole network than those in the quiet region. This atmospheric model can satisfactorily explain the distribution of emission measurements (EM) obtained from the EUV observations for $T > 10^5$ K. In addition it is also found that the Alfvén-wave heating at the base of the transition region of a coronal hole may exceed the heat conduction. The convection energy losses could be important in a coronal hole atmosphere due to the wave momentum deposition. In the transition region at heights greater than 650 km above the base of the transition region the convection energy losses may gradually exceed the radiation energy losses.

I. INTRODUCTION

Many investigators have paid attention to the energy balance in the chromosphere-corona transition region, because it relates to the heating of both the chromosphere and the corona. The main difficulty of any theoretical model for the transition region is the identification of the major energy terms. There is no doubt that all existing models included radiation and heat conduction. However, heat conduction is strongly affected by the magnetic field geometry. Therefore, the models of Gabriel,$^1$ Wragg and Priest,$^2$ and Athay$^{3,4}$ considered the magnetic field structure in their improved models based on spherical symmetry. In the former models,$^3,5,6$ either convection energy loss or mechanical-wave heating was usually neglected. This approximation may be acceptable for the active or the quiet region, but, as far as the corona atmosphere is concerned, the heat conduction flux is one order of magnitude smaller than that in the quiet region, as pointed out by Withbroe$^7$ based on a semi-empirical model developed from observations. If so, the downward heat flow can not balance the radiation loss in the transition region, implying the need for mechanical-wave heating. In addition, since the high-speed wind is dependent on the altitude in the corona,$^8,9$ the convection energy loss in the coronal hole can not be neglected. Furthermore, Munro and Jackson$^{10}$ derived from experimental data the cross section of the corona hole at $3R_\odot$ to be seven times that in the radial direction, indicating the dispersion of the magnetic field. Based on these observations, this paper attempts to propose an energy-balance model including nonradial distributions of mechanical-wave heating and convection energy loss. The result of the calculation is also compared with the observed data.

II. EQUATIONS AND BOUNDARY CONDITIONS

The magnetic field of the coronal hole consists of open field lines, and the field geometry is determined through the coupled equation

$$\nabla \cdot B = 0,$$  \hspace{1cm} (1)

and the equation for the coronal hole area (see Wragg and Priest$^2$),
A = A_0 \left[1 + \frac{a(R - R_\odot)}{R_\odot}\right]^2, \tag{2}

where \(A_0\) is the base area of the coronal hole, \(a\) the dispersion factor. When \(a = 0\), the corresponding atmosphere has a plane-parallel distribution.

With the interaction between the wave and the medium, the equation of motion for the average flux is\(^\text{11}\)

\[
\rho \left( \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \mathbf{U} = -\nabla \left( p + \frac{B^2}{4\pi} \right) + \frac{(B \cdot \nabla) B}{4\pi} - \nabla \cdot \mathbf{P}_w + \rho g, \tag{3}
\]

where \(g = -GM_\odot R/R^2\), \(\rho\), \(\mathbf{U}\), \(p\) and \(B\) are the average density, velocity, pressure and field strength of the fluid. \(M_\odot\) is the solar mass, and \(\mathbf{P}_w\) is the wave pressure tensor, representing the average effect of the wave on the fluid. For the Alfvén wave,\(^\text{12}\)

\[
\mathbf{P}_w = W \mathbf{k}k - \frac{1}{2} \mathbf{W} \tilde{I}, \tag{4}
\]

where \(W\) is the wave energy density, \(\mathbf{k}\) the wave vector, and \(\tilde{I}\) the unit tensor. The energy equation is

\[
\frac{\partial (E + W)}{\partial t} + \nabla \cdot \left[ \mathbf{Q} + W \mathbf{V}_g + \mathbf{P}_w \cdot \mathbf{U} \right] = 0, \tag{5}
\]

where \(E\) is the internal energy density (including the magnetic energy), \(\mathbf{Q}\) the energy flow and \(\mathbf{V}_g\) the group velocity of the wave front of the wave disturbance, i.e.,

\[
E = \frac{1}{2} \rho \mathbf{U}^2 + \frac{p}{\gamma - 1} + \frac{B^2}{8\pi},
\]

\[
\mathbf{Q} = \left( \frac{1}{2} \rho \mathbf{U}^2 + \frac{\tau p}{\gamma - 1} - \rho \frac{GM_\odot}{R^2} \right) \mathbf{U} - \frac{(\mathbf{U} \times \mathbf{B}) \times \mathbf{B}}{4\pi} + \mathbf{Q}_\eta, \tag{6}
\]

where \(\mathbf{Q}_\eta\) is the contribution from the radiation and heat conduction processes to \(\mathbf{Q}\). \(\gamma\) is the ratio of specific heats.

The conservation of wave-function density is

\[
\frac{\partial}{\partial t} \left( \frac{W}{\omega'} \right) + \nabla \cdot \left( \mathbf{V}_g \frac{W}{\omega'} \right) = -\frac{\mathbf{V}_g W}{d \omega'}, \tag{7}
\]

where \(\omega'\) is the critical frequency for the wave disturbance, and \(d\) the decay length.

The equation of continuity is

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{8}
\]

and the equation of state

\[
p = 2Nk_B T \tag{9}
\]

assuming an ideal ionized gas. \(N\), \(T\) are the density and temperature of the electrons, respectively, and \(k_B\) is the Boltzmann constant.

Consider the steady-state flow (\(\partial / \partial t = 0\)) along the radial direction in a tube with a cross-sectional area \(A(R)\). Since \(k\), \(B\), and \(U\) are all along the radial direction, the continuity equation becomes

\[
\frac{1}{A} \frac{d}{dR} \left( A \rho U \right) = 0. \tag{10}
\]
The equation of motion is now

\[ \rho U \frac{dU}{dR} + \frac{dp}{dR} + \frac{1}{2} \frac{dW}{dR} + \frac{W}{A} \frac{dA}{dR} - \frac{\rho G M_\odot}{R^2} = 0. \]  \(11\)

The energy equation of (5) becomes

\[ \frac{1}{A} \frac{d}{dR} \left\{ A \left[ \left( \frac{1}{2} \rho U^2 + \frac{5 \rho k_B T}{m_H} - \frac{\rho G M_\odot}{R} \right) U + W \left( V_A + \frac{3}{2} U \right) + F_c \right] \right\} = -N^2 L(T), \]  \(12\)

where \(V_A\) is the group velocity of the Alfvén wave,

\[ V_A = \frac{B}{\sqrt{4\pi \rho}}. \]  \(13\)

The conducting flux is

\[ F_c = -K_c \frac{dT}{dR}. \]  \(14\)

where \(K_c\) is taken from the result of Vanbeveren and De Loore,\(^{13}\)

\[ K_c = K_0 T^{5/2}, \]  \(14a\)

where \(K_0 = (1.89 \times 10^{-5})/\ln \Lambda\) in cgs units.

When \(T < 4.2 \times 10^5\) K,

\[ \Lambda = 3(2\pi)^{-1/2} \frac{(k_B T)^2}{e^2 \rho^{1/2}}. \]  \(14b\)

When \(T > 4.2 \times 10^5\) K,

\[ \Lambda = 3(2\pi)^{-1/2} \frac{(k_B T)^2}{e^2 \rho^{1/2}} \left( 4.2 \times 10^5 \frac{T}{T} \right)^{1/2}. \]  \(14c\)

\(N^2 L(T)\) is the radiation energy, and \(L(T)\) is taken from Ref. 14:

\[ \begin{array}{ll}
10^{4.3} < T \leq 10^{4.6}\text{ degree}, & L(T) = 10^{21.85}. \\
10^{4.6} < T \leq 10^{4.9}\text{ degree}, & L(T) = 10^{23.51} T^2. \\
10^{4.9} < T \leq 10^{5.4}\text{ degree}, & L(T) = 10^{24.21}. \\
10^{5.4} < T \leq 10^{5.75}\text{ degree}, & L(T) = 10^{23.4} T^{-1}. \\
10^{5.75} < T \leq 10^{6.3}\text{ degree}, & L(T) = 10^{21.95}. \\
10^{6.3} < T \leq 10^{7}\text{ degree}, & L(T) = 10^{22.73} T^{-2/3}. \\
\end{array} \]  \(15\)

The conservation of the wave function density of (7) becomes

\[ \frac{d}{dR} \left[ \mathcal{A}(V_A + U)^4 W / V_A \right] = -A(V_A + U)^3 W / (V_A \cdot d), \]  \(16\)

assuming that \(k\) and \(U\) are parallel, that there is no dispersion, and the WKB approximation is used. When \(U\) is much smaller than \(V_A\), the decay length can be approximated as\(^{12}\)

\[ d = 2.44 \times 10^{12} BN^{-1} T^{1/2}. \]  \(17\)

By solving Eqs. (9), (10), (11) and (12) simultaneously, one obtains
\[
\frac{dT}{dR} = -\frac{F_c}{K_c(T)},
\]

\[
\frac{dN}{dR} = \left[ \frac{W}{2d} - \frac{2a}{R_o} \left( W - U^2N m_H (1 + a f(R))^{-1} \right) - 2K_c N \frac{dT}{dR} - \frac{G N m_H M_\odot}{R^4} \right]
\]

\[
\left[ 2K_b T - U^2 m_H + \frac{W (V_A + 3U)}{4N (V_A + U)} \right],
\]

\[
\frac{dW}{dR} = -4K_b N \frac{dT}{dR} - \frac{2N m_H G M_\odot}{R^2} - \frac{4a}{R_o} \frac{W}{[1 + a f(R)]} + \frac{4a}{R_o} \frac{U^2 N m_H}{[1 + a f(R)]}
\]

\[
- (4K_b T - 2U^2 m_H) \frac{dN}{dR},
\]

\[
\frac{dF_c}{dR} = -N^4 L(T) - 5K_b N U \frac{dW}{dR} - \frac{G N m_H M_\odot U}{R^4} - (1.5U + V_A) \frac{dW}{dR}
\]

\[
+ \left[ m_H U^3 + \frac{3W U}{2N} + \frac{W V_A}{2N} \right] \frac{dN}{dR} + 2a \frac{N m_H U^3 - F_c}{R_o} \frac{1}{[1 + f(R)]},
\]

\[
U = \frac{\Phi_0}{N} \frac{1}{[1 + a f(R)]^{-2}}, \text{ where } f(R) = \frac{R - R_o}{R_o},
\]

where the Boltzmann constant \( K_b = 1.38 \times 10^{-16} \) erg·K\(^{-1}\), hydrogen atomic mass \( m_H = 1.67 \times 10^{-24} \) g, force constant \( G = 6.67 \times 10^{-8} \) dyne-cm\(^2\)/g\(^2\), solar mass \( M_\odot = 1.99 \times 10^{33} \) g, and solar radius \( R_\odot = 6.96 \times 10^{10} \) cm.

To normalize the set of equations above, we use

\[
R = R_\odot = H; \quad \frac{H}{H_*} = \frac{R}{R_*}; \quad \frac{T}{T_*} = \frac{N}{N_*}; \quad \frac{W}{W_*} = \frac{F}{F_*}; \quad \frac{F_c}{F_c_*} = \frac{F_c}{F_c_*},
\]

(22a)

to obtain

\[
\frac{dT}{dH} = 10^9 \frac{F_c}{K_c(T)},
\]

\[
\frac{dN}{dH} = \left\{ 4.80 \times 10^{-17} N U^2 a (1 + 1.44 \times 10^{-3} a H)^{-1} - 2.22 \times 10^4 N (H + 696)^{-2}
\]

\[
- 2.76 \times 10^{-2} N \frac{dT}{dH} - 2.88 \times 10^{-6} a W (1 + 1.44 \times 10^{-3} a H)^{-1} + 5 \times 10^4 \frac{W}{d}
\]

\[
\right\} / \left\{ 2.76 \times 10^{-2} T - 1.67 \times 10^{-4} U^2 + 2.50 \times 10^{-4} \frac{W (V_A + 3U)}{N (V_A + U)} \right\},
\]

\[
\frac{dW}{dH} = -5.52 \times 10 N \frac{dT}{dH} - 4.44 \times 10^7 N (H + 696)^{-2} + 9.60 \times 10^{-14} a N U^2 (1
\]

\[
+ 1.44 \times 10^{-3} a H)^{-1} - 5.75 \times 10^{-3} a W (1 + 1.44 \times 10^{-3} a H)^{-1}
\]

\[
- (5.52 \times 10 T - 3.34 \times 10^{-11} U^2) \frac{dN_c}{dH^*},
\]

\[
\frac{dF}{dH} = -10^{33} N^4 L(T) + 6.90 \times 10^{-7} U N \frac{dT}{dH} + 2.22 \times 10^{-1} (H + 696)^{-2} U N
\]

\[
+ (1.50 \times 10^{-8} U + 10^{-8} V_A) \frac{dW}{dH} - (1.67 \times 10^{-19} U^3 + 1.50 \times 10^{-8} W U N^{-1}
\]

\[
+ 5 \times 10^{-8} V_A W N^{-1}) \frac{dN}{dH} - (1 + 1.44 \times 10^{-3} a H)^{-1} (4.80 \times 10^{-22} a N U^3
\]

\[
+ 2.87 \times 10^{-8} a F_c),
\]

(26)
\[ U = 10^2 \Phi_0 N^{-3} (1 + 1.44 \times 10^{-3} a H)^{-2}, \]  
\[ d = 2.44 \times 10^{5} B_0 N^{-1.5} T^{1.5} (1 + 1.44 \times 10^{-3} a H)^{-2}, \]  
\[ V_A = 6.55 \times 10^4 N^{-0.5} (1 + 1.44 \times 10^{-3} a H)^{-2}. \]

From these seven equations (23)-(29), one can calculate the distributions of \( T, N, W, F_c, U, d, \) and \( V_A \) as functions of the altitude \( H \) (\( H = r - r_0 \)). In these equations, the bar for the normalized quantities was neglected for simplicity. The critical parameters for normalization were \( H_\ast = 10^8 \) cm, \( T_\ast = 10^4 \) K, \( N_\ast = 10^{10} \) cm\(^{-3} \), \( W_\ast = 10^{-3} \) erg cm\(^{-3} \) and \( F_{c\ast} = -10^5 \) erg cm\(^{-2} \) s\(^{-1} \).

Before proceeding further on the coronal-hole network model, we will discuss the effect of boundary conditions by taking the dispersion factor \( a \) to be 1.

We will first select a set of boundary conditions \( H_0 = 2 \times 10^8 \) cm, \( T_0 = 4 \times 10^4 \) K, \( N_0 = 1.8 \times 10^{10} \) cm\(^{-3} \), \( B_0 = 3 \) G, \( W_0 = 2 \times 10^{-3} \) erg cm\(^{-3} \) and \( F_{c_0} = -3.6 \times 10^5 \) erg cm\(^{-2} \) s\(^{-1} \). and \( \Phi_0 U_0 = 10^{13} \) cm\(^{-2} \) s\(^{-1} \) and calculate a theoretical model, (for convenience, the cgs units will be used throughout, and will not be repeated). Then, while keeping other boundary conditions constant, \( T_0 \) will be taken as \( 3 \times 10^4, 2 \times 10^4, \) and \( 10^4 \) for the calculation. It was found that when \( T_0 \) dropped from \( 4 \times 10^4 \) K to \( 10^3 \) K, both electron temperature and density were reduced. At \( 7.5 \times 10^9 \) cm altitude, \( T \) dropped 12\% and the \( N \) value was reduced by a factor of three.

If the other was kept constant and \( N_0 \) was varied as \( 1.2 \times 10^{10}, 0.6 \times 10^{10}, \) and \( 0.3 \times 10^{10} \), it was found that the reduction of \( N_0 \) also caused reductions in electron temperature and density. When \( N_0 \) was dropped from \( 1.8 \times 10^{10} \) to \( 0.3 \times 10^{10} \), \( T \) dropped 13\% and \( N \) was reduced by a factor of 4.5 at \( H = 7.5 \times 10^9 \) cm.

When \( W_0 \) was varied as \( 10^{-3}, 10^{-2}, 3 \times 10^{-2}, \) and \( 5 \times 10^{-2} \) for the calculation, the result showed that for \( H = 2 \times 10^8 \) cm \( T \) was reduced as \( W_0 \) was increased, whereas \( N \) increased with increasing \( W_0 \). For instance, at \( H = 3 \times 10^8 \) cm, \( T \) dropped 9\% and \( N \) increased 16\% over the range of \( W_0 \) variation. At \( H = 5 \times 10^8 \) cm and higher altitudes, \( T \) and \( N \) were almost constant as \( W_0 \) was changed.

When \( B_0 \) was set to be 1, 10, and 15 gauss, it was found that \( T \) and \( N \) increased with increasing \( B_0 \), but only by a few percent in this range of \( B_0 \).

When we varied \( a \) from 0.5 to 10, it was found that \( T \) was reduced but \( N \) increased with increasing \( a \). At \( H = 10^{10} \) cm, \( T \) decreased 29\% and \( N \) increased 18\% over this range of variation in \( a \).

### III. RESULTS AND DISCUSSION

The calculation showed that the atmospheric model varied with the boundary conditions \( T_0, N_0, B_0, \) and \( a \). In order to confirm the coronal-hole network model, one needs to seek support from observation data. We have selected the distribution of emission measurement EM in the coronal hole network obtained by Raymond and Doyle through EUV observation. From Ref. 15, EM was defined as

\[ EM = \int_{0.43T}^{1.12T} N^2 \sin H. \]

It is seen from this formula that EM is determined by the electron density \( N \) and temperature \( T \) in the atmosphere, with the former being the more sensitive variable. The calculation showed that multiple increases of the boundary values \( T_0 \) and \( N_0 \) cause the EM distribution to increase systematically. Similar increases in \( B_0 \) did not cause the EM distribution to move systematically, except for an increase in EM near the high temperature end, which changed the slope of the curve. Increases in \( a \) only caused the EM value at \( T = 10^6 \) K to decrease slightly. For the case of

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$H_0 = 2 \times 10^8$, $T_0 = 2 \times 10^4$, $N_0 = 1.3 \times 10^{10}$, $W_0 = 5 \times 10^{-3}$, $F_\infty = -6 \times 10^4$, $\Phi_0 = 5 \times 10^{13}$ and $B_0 = 5$, the calculated EM distribution agreed with the observed results satisfactorily (see Fig. 1). The coronal-hole network model is therefore confirmed as shown in Table I and Fig. 2. It should be pointed out that this is only for the range of $T > 10^5$ K. The distributions of $T$ and $N$ for the quiet region are also given in Fig. 2 for comparison.

We also calculated the energy balance in the network, as shown in Fig. 3 for $2 \times 10^4$ K < $T < 10^6$ K. It was found that the effect of the Alfvén-wave heating near the lower 90 km of the transition region was, together with heat conduction, to increase the transition atmosphere by more than one order of magnitude. Above 90 km, the effect of the Alfvén wave disappeared, and the atmosphere was only heated through heat conduction. In the lower 650-km range of the transition region, the atmosphere was mainly cooled through radiation, and the convection energy loss prevailed. Therefore, the convection-energy loss in a coronal hole network cannot be neglected.

The calculated results showed that the assumption of nonradial distribution and the inclusion of wave heating in the energy balance led to a theoretical atmospheric model that explains the EM distribution from the EUV observation. The dispersion factor of such a flow tube is $a = 3.5$, and the predicted coronal-hole network area at $3R_\odot$ is about 7 times that for the radial case. In the transition region, the area is similar to that for the radial case, in agreement with the observation of Munro and Jackson.\textsuperscript{10}

The electron density in the coronal-hole network is half that in the quiet region, and the temperature is 60% cooler. The thickness of the transition region is four times that of the quiet region. These results agree with the observation of the SkyLab.\textsuperscript{16}

The atmospheric pressure $p$ in the coronal-hole network is one third that in the quiet region, and decreases much more rapidly than in the quiet region. In the altitude range of $2 \times 10^8$–$10^{10}$ cm, the pressure $p$ dropped from $7.2 \times 10^{-2}$ dyne/cm$^2$ to $2.0 \times 10^{-2}$ dyne/cm$^2$, showing that the pressure is not a constant in the transition region.

Based on the WKB approximation and the atmospheric model, it was predicted that the Alfvén frequency is only a fraction of a hertz, several times less than that in the quiet region.
These waves decayed during propagation in the 90-km range in the transition region of the coronal-hole network, and hence heated the atmosphere. Especially in the thin layer of 10 km range, the heating effect of such a decay has exceeded heat conduction. Therefore, Alfvén-wave heating is important in a low temperature, low-density coronal hole atmosphere.

Our theoretical model still does not explain the EM distribution in the $T < 10^5$ K range. It is possible that there exists a needle-shaped radiation pattern, or the radiation is not mainly from the open magnetic flux tube. As Dowdy\textsuperscript{17} pointed out, this portion of the radiation could
be from the low, closed magnetic flux tubes. The field along these closed tubes strongly prevents the flow and the heat flux, such that the internal gas is effectively insulated from the corona and the $T > 10^5$ K thermal transition region. Therefore, this cold ($T < 10^5$ K) transition region has to be heated internally, not by the heat from above. These low, closed magnetic flux tubes exist in the quiet region, and may also exist in the transition region.

The boundary values for our coronal hole network are selected, on one hand to satisfy the observed data, and on the other hand based on observation. For instance, the field $B_0$ was taken to be 5G, close to 3G in the quiet region. Reference 16 pointed out that the field strengths in the coronal hole and the quiet region are similar. $T_0$ was taken to be $2 \times 10^4$ K and $N_0 1.3 \times 10^{10}$ cm$^{-3}$ (slightly less than in the quiet region), in agreement with the observed spectral strength of the transition region in the coronal hole and the quiet region. The selection of $U_0$ to be $3.77 \times 10^3$ cm/s was based on the solar-wind observation and extrapolation of Chiuderi of $U_0 = 4.49 \times 10^3$ cm/s for the transition bottom and the conclusion of Pneuman et al. that there is a steady flow at a few kilometers per second in the transition region above the chromosphere network.


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