ON THE LENGTH OF THE SYNODIC MONTH

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As is well known, the mean length of a lunation is a little more than 29.5 days (more accurately, 29.5306 days). However, the actual length of a lunar month can vary considerably about this mean from one lunation to another. Lunar calendars, both ancient and those still in use today (e.g. Babylonian, Chinese, Hebrew, and Islamic), assume month lengths of 29 and 30 days. On each calendar there is a slight excess of 30-day months so that the average is close to 29.5306 days. From time to time, we have received enquiries from students of these calendars regarding the range in the true length of a lunar month, and this has prompted us to undertake an investigation of the degree of variability. Here, we first illustrate the short-term variation in the length of a synodic month (over a selected period of two years) and subsequently deduce the maximum and minimum lengths of a lunation over a period of several millennia.

The length of a lunation—i.e., the interval between two successive conjunctions in longitude of the Moon and the Sun—is controlled by a variety of factors. Principal among these is the fact that the mean synodic month is about two days longer than the sidereal month. As a result, the Moon makes rather more than one sidereal revolution in the course of a lunation. Both the terrestrial and lunar orbits are appreciably elliptical. In consequence, the apparent motion of the Sun during a given lunation and the angular velocity of the Moon during the corresponding roughly two-day excess period are the main factors in determining the precise length of a lunation. The principal variation in month-length is a function of the true anomaly of the Moon at conjunction. If conjunction occurs when the Moon is near perigee, month-lengths are slightly shorter than average; near apogee, the opposite is the case. A lesser effect depends on the angular distance of the Earth from perihelion. The result is for longer month-lengths to occur when conjunction is near perihelion and for shorter lengths near aphelion.

As an illustration of the variation in the synodic month in a roughly two-year period, we have computed the length of each lunation in the interval from early 1990 to early 1992 (Fig. 1). In that interval the length of a lunation varied from approximately 29°.30 to 29°.83 days.

These variations are responsible for complexities in the operation of a lunar calendar. For example, the Islamic calendar mainly uses alternate months of 30 and 29 days; discrepancies between this calendar and an ideal calendar based on the Moon itself can often exceed one day. The Chinese calendar approximates more closely to a truly lunar calendar. Near the peak of a month-length cycle up to four 30-day months can follow one another, while at the opposite phase two 29-day months can succeed one another. The maximum discrepancy between this calendar and one keeping accurate pace with the Moon is never more than one day.
As part of a project involving the study of lunar syzygies, we have computed the date and time of every New and Full Moon in the period from 1000 B.C. to A.D. 4000. From these results, we have derived the lengths of each lunation during this interval and hence the maximum and minimum lengths of a synodic month in this entire period. The absolute maximum length proved to be 29.8376 days (in 490 B.C.), while the extreme minimum length was 29.2679 days (in 302 B.C.). The equivalent total range is 0.5697 days or 13 hours, 40 minutes.

The following temporal effects seem worth commenting upon. Firstly, there is a gradual forward drift in the date of perihelion. As a result, although in the ancient past the longest months always occurred in December, by medieval times this had changed to January — as it still is today. By about the year 2200, the longest months will begin to occur in February. Again, in ancient times the extreme minimum month-lengths always occurred in June (near aphelion), but in the medieval period this had changed to July — again, as is the case at the present day. By about 2300, the shortest months will begin to take place in August.

It is also worth pointing out that the length of the mean synodic month has not changed significantly during the historical period. Due to the action both of tides and the decreasing eccentricity of the Earth's orbit (with consequent change in solar pertubations), the lunar motion is gradually being decelerated. The combined effect is to cause a slow increase in the length of the mean synodic month, by only some 0.2 seconds per millennium. Over the 5,000-year period of our survey we have noted a small secular decrease in the maximum length of months and a corresponding increase in the minimum month-lengths. This arises from the gradual decrease in the eccentricity of the Earth's orbit due to planetary action. Linear fits to our computed extreme month-lengths for each century show a gradual decrease in the maximum and a gradual increase in the minimum by close to 0.0020 days (about 3 minutes) per millennium.

As far as the maintenance of a reliable lunar calendar is concerned, the secular variation in the length of the month is purely an academic issue. However, it is evident that not all major lunar calendars make adequate allowance for the annual variations in the length of the month.

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