THE RESEARCH OF THE TIME VARIATION OF H$_2$O MASER

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Abstract. To explain the intensity fluctuations observed from bursts of W3(OH) and Cep A H$_2$O masers in the period of 50–60 days, we propose a new idea that energy of the maser pump is injected in the maser region with pulse form not only once but several times, and that it is supplied by the radiation diffusing through the gas cloud, in spite of the pump mechanism being radiation pump or collision pump. With our model the fits are obviously better than what many authors obtained in previous literature.

1. Introduction

After the lines of interstellar water molecules were discovered, it was soon found that the remarkable features of H$_2$O maser in the regions of the star formation are the time-variations of their radiation. This fact of observation perhaps reflects the turbulence associated with the outflow in the regions of protostar or newly-born star. Therefore, H$_2$O maser may be the probe of star-formation regions; we shall understand the correlation between H$_2$O maser and star formation and learn more information about star formation through the detailed research in the time variation of H$_2$O maser.

The earliest work about the time-variation of H$_2$O maser was carried out by Sullivan (1973). He found that the time-scale of variation could be about one week, and also found that the time variations existed in the different features, linewidth and strength. Through further research, it was also found that relative time variations occurred among individual masers in the same region (Gammon, 1976; White, 1979). Some observations showed that the variations could occur at any time. These proved that some variations were very rapidly and processed short period (White and Little, 1975; Lo et al., 1976).

Because of the time-variations of H$_2$O maser, especially the rapidly time-varying characteristics processed by some sources, many observations and theoretical investigations have been devoted to these interesting topics (Little et al., 1977; Abraham et al., 1981; Matveenko, 1986; Sandell and Olofsson, 1981). The observational results of W3(OH) showed that emission of H$_2$O maser at 1.3 cm wavelength increased very rapidly during a short time, and reached its maximum value soon thereafter; then declined smoothly (Haschick et al., 1977). The same result had been obtained from the observations of Cep A H$_2$O maser (Mattila et al., 1983). In order to explain this features of time-varying H$_2$O maser, Burke et al. (1978) put forward a simple model, in which he assumed that the energy of the maser pump was supplied by the radiation diffusing...
in the gas cloud, while the pump mechanism itself might work through either radiation or collision. Burke's model was rather successful in explaining the time variations of $W_3(OH)H_2O$ maser. With the modified Burke's model, Mattila et al. (1983) interpreted the time variations of Cep A $H_2O$ maser. In the modified model, they assumed a linear maser geometry, a fully-saturated maser which was collisionally pumped by the heated gas, and the gas temperature was proportional to the radiation density. They also fitted the observations well.

Although Burke's and Mattila's models can explain the observational data well, it was difficult for observations to be explained in terms of these models - such as the obvious intensity fluctuations in the data. At these sharp peaks the value calculated from the models could not fit the observational data. Burke et al. (1978) thought this was caused by the observational errors. But because not only in $W_3(OH)$ but also in Cep A the same phenomena occurred, we considered that these must be caused by some physical processes in the regions.

To explain these intensity fluctuations observed from $W_3(OH)$ and Cep A $H_2O$ masers, we modify Burke's model further and propose a new idea that the energy of maser pump is pumped in the maser region with pulse form not only once but several times. On the basis of our model, the calculated results can fit the observational data better.

### 2. Physical Model

We assume that the energy for the $H_2O$ maser pump is supplied by the radiation diffusing through the gas cloud, in spite of pump mechanism being radiation pump or collision pump. Therefore, the luminosity-$L_{ph}$ of maser source for a spherical, isotropic and saturated maser, will be given by

$$L_{ph} = 4\pi\varepsilon n_{H_2} \int_0^\infty \Delta R(r)r^2 dr,$$

(1)

where $R$ is the difference in pump-rate per water molecule to the upper and lower masing levels, $n$ is the number density of hydrogen molecules. We postulate that the radiative energy diffuses through a uniform gas cloud characterized by a diffusion constant $D$, and assume that approximate thermalization is quickly attained. We also assume that the energy imported is a pulse energy to which we approximate by a $\delta$-function in space and time. Besides, the process of importing energy will not occur only once but several times within some interval times. After each energy input, the energy will diffuse gradually with the time. It is the diffusing energy that causes the pump rate of $H_2O$ maser varying with time and, therefore, the radiation strength of maser varies with the time. So generally, the energy density for the molecular cloud with diffusion constant $D$ can be expressed as

$$U(r, t) = E_0(4\pi D t)^{-3/2} \exp\left(-r^2/4Dt\right) +$$

$$+ \sum_{i=1}^n E_i[4\pi D(t - T_i)]^{-3/2} \exp\left[-r^2/4D(t - T_i)\right],$$

(2)
where $E_i$ is the value of injected energy with pulse form at the $(i + 1)$th time, $T_i$ is the time-lag of the $(i + 1)$th energy input, and $E_1$ is the value of the first energy injection.

Considering the thermal energy and radiative energy in molecular clouds, we obtain

$$U(r, t) = \frac{1}{2} f n_{\text{H}_2} k T + a T^4,$$

where $k$ is Boltzmann constant; $a$, the Stefan constant; $f$, the number of the degrees of freedom of hydrogen molecules. Under the conditions of molecular clouds, we shall only consider the freedom degrees of translation and rotation; and then $f$ equals 5. Because the temperature is only about 200 to 1000 K in our model, the term of $a T^4$ can be ignored. Therefore,

$$U(r, t) = \frac{1}{2} f n_{\text{H}_2} k T;$$

that is,

$$T = 2U(r, t)/(f n_{\text{H}_2} k).$$

In addition, the converse populations of H$_2$O maser levels are mainly caused by the collisions between hydrogen molecules and water molecules under the collisional pump-mechanism. But not all hydrogen molecules which take part in the collisions with the water molecules make contributions to the converse populations of maser energy levels. Only those H$_2$ of which the kinetic energy is high enough to pump the water molecules of lower levels to the upper maser level or the levels which are higher than it can contribute towards the maser pump. Therefore, the difference of pump rate $\Delta R$ is given by

$$\Delta R = n_{\text{H}_2} \langle v \rangle < v >_0 \rightarrow \infty \langle \Delta \sigma_c \rangle \langle g \rangle \exp(-T_{ex}/T)/Z(T),$$

where $Z(T) = (\pi/ABC)^{1/2} (k T)^{3/2}$ is the partition function; $ABC$ are molecular constants; $\langle g \rangle$ is an effective statistical weight; $\langle \Delta \sigma_c \rangle$ is a mean differential collisional cross-section; $\langle v \rangle_{v_0 \rightarrow \infty}$ is an averaged velocity of those hydrogen molecules whose kinetic energy $(1/2)m_{\text{H}_2} v^2_0 k(T_M - T_{ex})$ and made contributions to maser pump (assuming that all H$_2$O lie at the level of which the excitation energy is $kT_{ex}$, and $kT_M$ equals the excitation energy of H$_2$O maser upper level), that is

$$\langle v \rangle_{v_0 \rightarrow \infty} = \int_{v_0}^{\infty} v f(v) \, dv.$$

By Maxwell velocity distribution, we obtain

$$\langle v \rangle_{v_0 \rightarrow \infty} = (8kT/\pi m_{\text{H}_2})^{1/2} \left[ 1 + (T_M - T_{ex})/T \right] \exp[-(T_M - T_{ex})/T] =$$

$$= (8kT/\pi m_{\text{H}_2})^{1/2} f_c ,$$

where

$$f_c = \left[ 1 + (T_M - T_{ex})/T \right] \exp[-(T_M - T_{ex})/T].$$
Substitute Equation (8) into Equation (6), we can obtain
\[ \Delta R = n_{H_2} \langle v \rangle \langle \Delta \sigma_c \rangle \langle g \rangle \exp(-T_{ex}/T)f_c/Z(T), \]  
where
\[ \langle v \rangle = (8kT/\pi m_{H_2})^{1/2}. \]
Substitute Equations (2), (5), and (10) into Equation (1) which can be expressed as
\[ L_{ph}(\tau) = L_{ph}^0 B_1(\tau); \]
where
\[ B_1(\tau) = \int_0^\infty f_c \beta \exp(-\beta)x^2 \, dx, \]
\[ \beta = \tau^{3/2} \exp(x^2/\tau), \]
the dimensionless variables
\[ x = r/r_0, \quad \tau = t/t_0 \]
and
\[ t_0 = (1/4\pi D) (2E_0/f n_{H_2} k T_{ex})^{2/3}, \]
\[ l_0 = (1/\sqrt{\pi}) (2E_0/f n_{H_2} k T_{ex})^{1/3}; \]
as well as
\[ \Gamma = \left\{ 1 + \sum_{i=1}^n (E_i/E_0) [(\tau - T_i/t_0)/\tau]^{-3/2} \times \right. \]
\[ \left. \times \exp[-(T_i/t_0)x^2/\tau(\tau - T_i/t_0)] \right\}^{-1}, \]
\[ L_{ph}^0 = (8/\pi)^{3/2} (ABC/m_{H_2})^{1/2} \langle \Delta \sigma_c \rangle \langle g \rangle \in h_{H_2} E_0/f (k T_{ex})^2. \]
By means of Equation (1), (11) the number of photons emitted per second of the H$_2$O maser source can be calculated.

### 3. Results and Discussions

For typical H$_2$O maser region, the following working numbers are reasonable, those are: $E_0 = 0.879 \times 10^{14} n_{H_2} l_{ph}^{3/2}$ ergs (where $l_{ph}$ is the effective photon mean free path, and the value $l_{ph} = 10^{12}$ cm is adopted), $n_{H_2} = 0.5 \times 10^5$ cm$^{-3}$, therefore, $E_0 = 4.4 \times 10^{40}$ ergs, the diffusion coefficient $D = (1/3)c_{ph}$, $\langle \Delta \sigma_c \rangle = 10^{-17}$ cm$^2$, $\langle g \rangle = 50$, $\varepsilon = 3 \times 10^{-5}$, $T_{ex} = 200$ K, $T_M = 645$ K. We have calculated Equation (1). The results are showed in Figures 1 and 2, and the comparisons between the calculated results and the data of W3(OH) and Cep A are also showed in Figures 1 and 2 separately.
Fig. 1. Ratio of the peak antenna temperature of the $-50.4$ km s\(^{-1}\) feature in the H\(_2\)O spectrum of W3(OH) with respect to that of the $-48$ km s\(^{-1}\) feature plotted as a function of time from 1977 May 8 to June 16. The curve is our calculated result. + is observational result (Burke et al., 1978).

(1) For Cep A, we adopt a model in which the energy is composed of nine times the energy injected in pulse forms. It can be seen that the fit is remarkably good, except for only one point.

(2) For W3(OH), a model with twice the imported energy with pulse form is adopted. If more energy is injected, the fit becomes better.

(3) On the basis of a good fit of the data, the number of imported pulse energy should be as few as possible.

(4) Now the monitors of rapidly time-varying H\(_2\)O maser and theoretical research

Fig. 2. The peak antenna temperature of the $-11.2$ km s\(^{-1}\) feature in H\(_2\)O outburst of Cep A. The curve is our calculated result. + is observational result (Mattila et al., 1985).
of these sources are not sufficient. If we do more observations, the more rapidly time-varying H₂O maser sources may be discovered.

(5) We have not understood the mechanism of H₂O maser pumping very well, especially about the fluctuations. Some investigators think that the time variation of radial velocity in the H₂O maser may be caused by Doppler motion (Lekht et al., 1982). If so, the relationship between the fluctuations and the Doppler motion should also be considered further.

References