MAGNETIC FIELD ESTIMATION IN MICROWAVE RADIO SOURCES

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Abstract. Eliminating the term NL, useful formulae for the magnetic field estimation in microwave radio sources are presented. Applications of these formulae to observed solar microwave radio bursts are shown.

From the theoretical point of view, microwave spectroscopy should have the potential ability to diagnose the behaviour of energetic electrons and magnetic fields in radio sources, but either additional assumptions or data are usually necessary to separate the properties of the magnetic field from those of the electron distribution. In fact, it seemed impossible to determine the magnetic field from the gyrosynchrotron or synchrotron radiations until simplified relations for gyrosynchrotron radiation were found by Dulk and March (1982). Since then several papers (e.g. Batchelor et al., 1984; Dulk, 1985; Gary, 1985; Tandberg-Hanssen and Emslie, 1988; Bastian and Gary, 1992; Lira et al., 1992), which estimate the magnetic field on the basis of these relations, appeared, all using several additional assumptions. In this paper, we decrease the number of these assumptions and thus present a more direct way to estimate the magnetic field in microwave sources.

Let us assume that the radio source is homogenous along the line of sight, i.e. the vector magnetic field B and the number density N of superthermal electrons are constant along this line throughout the depth of radio source L. As was done by many other authors (e.g. Dulk, 1985; Tandberg-Hanssen and Emslie, 1988), we start from the condition for the microwave peak frequency expressed as

\[ \kappa_{\nu_{\text{peak}}} L \approx 1, \]

where \( \kappa_{\nu_{\text{peak}}} \) is the absorption coefficient at peak frequency \( \nu_{\text{peak}} \). Because it is convenient to write the absorption and emission coefficients in the form of \( \kappa_{\nu} B/N \) and \( \eta_{\nu}/(BN) \), let us rewrite relation (1) to

\[ \left( \frac{\kappa_{\nu_{\text{peak}}} B}{N} \right) \frac{NL}{B} \approx 1. \]

To proceed further we need to express the term NL, which is the total number of superthermal electrons along the line of sight \( N_{\text{tot}} \). This term is usually estimated under some assumptions about the radio source. But, at this point we continue in a new way. For the optically thin radio source, i.e. for frequencies \( \nu \gg \nu_{\text{peak}} \) we can write

\[ T_b = \frac{c^2}{k\nu^2} \eta_{\nu} L, \]

where $T_b$ is the source brightness temperature, $c$ is the speed of light, and $k$ is the Boltzmann constant. As in Equation (2) we rewrite this relation in the form

$$T_b = \frac{c^2}{\kappa_\nu B} \frac{\eta_\nu}{B N} B N L. \quad (4)$$

Expressing the term $N_{tot} = N L$ from this relation as

$$N_{tot} = N L = \frac{T_b k B^2}{\kappa_\nu c^2 B N} \quad (5)$$

and setting it into Equation (2) we obtain a general implicit formula for the dependence of the magnetic field on the peak frequency in the form

$$\left(\frac{\kappa_\nu B}{\kappa_\nu}\right) k B^2 \frac{T_b}{c^2 B^2} \approx 1. \quad (6)$$

It is important to emphasize that in this formula the magnetic field does not depend on frequency $\nu$. Namely, both the emission coefficient $\eta_\nu$ and the brightness temperature $T_b$ depend on $\nu$, but $T_b$ is proportional to $\eta_\nu \nu^{-2}$, thus eliminating the dependence on $\nu$ in Equation (6).

Now, let us apply this formula for two cases:

**a) The case of gyrosynchrotron radiation of mildly relativistic electrons.**

The emission and absorption for this case were studied by Dulk and March (1982) who found the following analytical approximations for the emission and absorption coefficients of the x-mode for $2 \leq \delta \leq 7$, $\theta \geq 20^\circ$, $\nu/\nu_B \geq 10$, and $E_0 = 10$ keV to be

$$\frac{\eta_\nu}{B N} \approx 3.3 \times 10^{-24} 10^{-0.525 (\sin \theta)^{-0.43} + 0.655 \left(\frac{\nu}{\nu_B}\right)^{1.22 - 0.905}}, \quad (7)$$

$$\frac{\kappa_\nu B}{N} \approx 1.4 \times 10^{-9} 10^{-0.225 (\sin \theta)^{-0.09} + 0.725 \left(\frac{\nu}{\nu_B}\right)^{-1.30 - 0.986}}, \quad (8)$$

where $\nu_B$, $\delta$, and $\theta$ are the gyrofrequency, the power-law energy index of electrons and the viewing angle, respectively.

Using Equation (6) with these expressions for emission and absorption coefficients, we can express the magnetic field as follows

$$B \approx \left(\frac{c^2}{kT_b A_1} \nu_{\text{peak}}^{1.30 + 0.986} \nu^{-0.78 - 0.905} A_2^{-2.52 - 0.086} \right)^{\frac{1}{3.35 + 0.585}}, \quad (9)$$

where

$$A_1 = 4.24 \times 10^{14} 10^{0.36 (\sin \theta)^{0.34} + 0.075} \quad (10)$$

and $A_2 = 2.8 \times 10^6 (\nu_B = A_2 B)$.

Since we know now the magnetic field, we can estimate the total number of superthermal electrons in the microwave source along the line of sight $N_{tot} = N L$ according to Equation (5).
b) The case of synchrotron radiation of relativistic electrons.

In the ultra-relativistic limit for the power-law distribution function of electrons, the emission and absorption coefficients (for each of the o- and x-modes) can be expressed (see Dulk, 1985, Lang, 1980)

\[
\frac{\eta_\nu}{B N} = \frac{1}{2}(\delta - 1)E_0^{\delta-1}g(\delta)\frac{30.5e^2}{8\pi mc^2}\sin\Theta\left(\frac{2m^2c^4 \nu}{3\sin\Theta \nu_B}\right)^{-\delta/2}, \quad (11)
\]

\[
\frac{\kappa_\nu B}{N} = (\delta - 1)E_0^{\delta-1}h(\delta)\frac{2\pi mc^5e^2}{9}\frac{1}{\sin\Theta}\left(\frac{m^2c^4 \nu}{3\sin\Theta \nu_B}\right)^{-\delta/4}. \quad (12)
\]

Using the formula (6) with these expressions for emission and absorption coefficients, the magnetic field in this case can be expressed as follows:

\[
B \approx \left(\frac{c^2}{T_b k\nu^2 A_2 A_3^2}\right)^2, \quad (13)
\]

where \(A_2 = 2.8 \times 10^6\) and

\[
A_3 = \frac{h(\delta)}{g(\delta)}\frac{32\pi^2 m^6 c^{12}}{3^2 e^2}\frac{1}{(\sin\Theta)^2}\left(\frac{m^2c^4 \nu_{peak}}{3\sin\Theta}\right)^{-\delta/2}\left(\frac{2m^2c^4 \nu}{3\sin\Theta}\right)^{(\delta-1)/2}. \quad (14)
\]

Now, let us apply the derived relations for the magnetic field estimations to recent observations of high-frequency radio bursts published by Lim et al. (1992). In this paper the radio emission of six solar flares at 5, 8.8, 15.4 and 86 GHz was studied. They found the peak (turnover) frequency at about of 15.4 GHz, and the power-law energy index of electrons as \(\delta = 2.5 - 3.6\). The brightness temperature at 86 GHz can be estimated to be \(T_b = 1 \times 10^7 - 5 \times 10^7\) K.

In accordance with this paper, the gyrosynchrotron radiation of mildly relativistic electrons is considered to be the mechanism for the generation of observed bursts. Therefore, in this case we can use Equation (9) for the magnetic field estimation. Results of these estimations are shown in Figure 1, where the magnetic field as a function of the power-law energy index \(\delta\) for three peak frequencies \(\nu_{peak} = 10, 15, 20\) GHz and for two brightness temperatures on 86 GHz \(T_b = 1 \times 10^7\) K (full lines) and \(T_b = 5 \times 10^7\) K (dashed lines) is depicted. The viewing angle \(\Theta\) is assumed to be 45°. Taking, for example, the peak frequency as 15.4 GHz, \(\delta = 2.7\) and \(T_b = 1 \times 10^7\) K (the parameters close to that of Flare 1 in the paper of Lim et al. (1992) the estimated magnetic field is \(B = 402\) G, which seems to be reasonable. Moreover, using Equations (5) and (7) the total number of superthermal electrons (with the energy \(E > E_0 = 10\) keV) along the line of sight can be estimated as \(N_{tot} = N L = 6.52 \times 10^{16}\) cm\(^{-2}\). Assuming the source depth \(L\) comparable with its size (\(\approx 5\) arc sec) we can estimate the density of superthermal electrons with energy greater than 10 keV in the radio source to be \(1.8 \times 10^8\) cm\(^{-3}\).

From Figure 1 it can be seen that the magnetic field is increasing with the peak frequency increase, but it is decreasing with the increase of the brightness temperature. This second dependence looks strange, but it can be explained in the following way: If
Fig. 1. The magnetic field as a function of the power-law energy index of mildly relativistic electrons for three peak frequencies $\nu_{\text{peak}} = 10, 15, 20$ GHz and two brightness temperatures on 86 GHz $T_b = 1 \times 10^7$ K (full lines) and $T_b = 5 \times 10^7$ K (dashed lines). The viewing angle $\Theta$ is assumed to be 45°.

The peak frequency is constant and if the brightness temperature at frequencies $\nu \gg \nu_{\text{peak}}$, where the radio source is optically thin, is increasing then the term in Equation (6) $\kappa \nu_{\text{peak}} / \eta_{\nu}$ must be decreasing and this decrease leads to the decrease of magnetic field.

References