Magnetic reconnection theory for coronal loop interaction

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Abstract. Images of post-flare loop systems recorded in coronal green- and red-line emissions display occasional transient enhancements at the intersection of some loops where they come into contact. Such enhancements are investigated in terms of the likely plasma processes involved in these dynamic events. For this, the interaction between magnetohydrodynamic and high frequency plasma waves, and the instability associated with an electromagnetic solitary wave in a current sheet, are studied. It is shown that there is a resistive instability, which eventually turns into an eruptive instability at the onset of magnetic field reconnection. The numerical results are consistent with the observations. Thus the phenomenon of occasional enhancements in the vicinity of the projected intersection of two loops may be basically interpreted by this theory of magnetic field reconnection driven by ponderomotive force.

Key words: magnetohydrodynamics (MHD) – plasmas – Sun: corona – instabilities

1. Introduction

High-quality observations of post-flare loop systems in visible emission lines reveal transient brightenings at the intersections of some overlapping loops, due apparently to localized loop interactions. The reconfiguration of magnetic fields is apparently fundamental to most solar dynamical processes. This can occur on both short (seconds) and long (hours) time scales, and at relatively high (e.g. flares) and at relatively low (e.g. some fast coronal events) energies. Interacting coronal loops have been investigated in recent years in relation to flare studies. For example, Rust & Somov (1984) found that observed X-ray brightening starts near the intersection of two flare loops. Further studies of transient X-ray brightenings of interconnecting loops in the early post-flare phase have also been carried out by Spicer & Svestka (1983), and by Svestka & Poletto (1984). Machado et al. (1988a, b) have found that the interaction of current loops is an essential ingredient in the trigger of the solar flare energy release. In a related study, Den & Somov (1989) pointed out that a large energy release can be achieved through magnetic reconnection in thin current sheets.

VLA 6-cm observations have provided evidence of interacting loops. The reconnection process accelerates electrons to energies ≥ 100 Kev, which gives rise to microwave bursts (Kundu 1983, 1985; Kundu et al. 1982, 1984, 1990). Another VLA measurement (Lang & Willson 1984) at 20 cm also gives observational evidence for flare build-up in coronal loops. These observations show that new bipolar loops can emerge and interact with pre-existing ones, thereby triggering impulsive bursts. Recent observations at decimeter and microwave frequencies have shown that millisecond spikes are a phenomenon associated with the impulsive phase of primary energy release in flares (Benz 1986). In a study of microwave and x-ray spikes produced by a small flare, Sakai & De Jager (1989, 1991) have suggested that associated small plasma knots can originate by x-type (3D) flux tube coalescence.

Tajima et al. (1982, 1985, 1987) and Sakai (1990a, b) have proposed a simultaneous acceleration mechanism of protons and electrons in a solar flare by the non-linear coalescence instability of two current loops based on the results of a computer simulation. Two parallel current loops are unstable against the coalescence instability (Pritchett & Wu 1979). If the currents have the same sense, the loops are attracted by and collide with each other and finally coalesce into one loop. Its non-linear development can release a large amount of magnetic energy, associated with the current loops, into particle energy. Nakajima et al. (1985) have shown that when two parallel loops have sufficient electric currents, they can attract each other fast enough (in about one Alfven transit time) such that the fast reconnection that takes place at the x-point (point of intersection of the loops) is disrupted as the plasma blobs overshoot. When two parallel loops have insufficient electric current or are well separated, the attractive force is weaker and reconnection becomes slower.

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In this paper, we investigate the subtle interaction between the magnetohydrodynamic (MHD) and high frequency plasma waves (plasmons), and examine the instabilities associated with electromagnetic solitary waves in a current sheet and show that there is a resistive instability, which eventually turns into an eruptive instability at the onset of magnetic field reconnection. We use this theory of reconnection to explain the phenomenon of occasional enhancements in the vicinity of the projected intersection of two loops. Section 2 provides a summary of the observations, followed by the basic theory in Sect. 3. Section 4 discusses instabilities caused by solitary waves. Computational results are given in Sect. 5, followed by some conclusions in Sect. 6.

2. Summary of observations

The data discussed here are from the 20-cm aperture NSO/SP emission-line coronagraph that records photographically sequential images in Fe XIV (5303Å), Fe X (6374Å) and Hα, with a normal cadence of one minute. We describe briefly the loop interaction features in post-flare loop systems (in more detail, see Smartt & Zhang 1990; Zhang & Smartt 1991; Smartt et al. 1993).

High resolution images in the green line show occasional marked enhancements in the vicinity of the projected intersection of two loops. From the data it is evident that such enhancements occur as one loop develops or moves to intersect a neighbor, producing an x-point of coalescence. The morphology of such events as seen at their maximum is a brightness at the point of intersection far greater than the sum of the brightnesses of the individual loops, and some enhancements extending partially along the loops away from the intersection point. Further, there is some partial “filling in” of the space between the loops, marked by a sharp boundary, evidently a process of redistribution of magnetic flux between interacting loops that results in partial magnetic reconnection. About ninety events have been observed. It is found that such enhancements increase to a maximum over a period of typically 10-15 minutes and then gradually fade with a lifetime of 20-30 minutes. It appears that this is a relatively common phenomenon as detected in the visible emission corona, especially around solar maximum with an average of at least several events per day. In a systematic effect, all such maxima are followed by corresponding maxima in red line (Fe X 6374Å) images with a lag on average of about 9 minutes. Approximately 9 minutes later again, Hα reaches a maximum at the intersection site.

Figure 1a-f illustrates typical events recorded in a flare-associated loop system of 7 December, 1981. This system contained many complex structures as indicated in the green-line images of 16:36 UT (a). A feature (A) on the right-hand side, with a height ~ 3 x 10^4 km, reaches a maximum brightness in green-line emission at 16:49 UT (b), while the corresponding (16:49 UT) red-line image (c) does not show this enhancement. By 16:56 UT, this enhancement (A) in the green-line image (d) has faded slightly, while the enhancement (B) on the left-hand side, at a height ~ 1.8 x 10^4 km, has become brighter and higher.

At the same time (16:56 UT), the red-line image (e) now shows the enhancement (A) at a maximum, a lag of 7 minutes from its maximum brightness in the green-line emission. By 17:04 UT, in the red-line image (f), the enhancement (A) has faded, while the enhancement (B) is now at a maximum, a lag of 8 minutes from its maximum in green-line emission.

It should be pointed out that a post-flare loop event is itself a highly dynamical system in which the loops, typically in a complex configuration, gradually increase in height. During this time period of several hours, new loops can appear, while others disappear, with a systematic trend to simpler configurations, a decreasing number of loops and fainter emission. Apparently it is this dynamical property of post-flare loops that strongly increases the probability of loop coalescence, as compared with coronal loops in general.

3. Basic theory

As suggested above, it appears that there is a subtle interaction between the MHD and the emission fields. On the other hand, according to the reconnection theory of solar flares, the thickness of the dissipation layer must remain at a very small value in order to provide sufficient power; but the hot plasma, resulting from the magnetic field dissipation, will tend to widen the current sheets (Pikel'ner & Kaplan 1978). The key is to find a local eruptive instability for which dissipation products can be thrown out at a fast enough rate. Of course, an instability with a small scale inevitably involves nonlinear interaction between the emission and the MHD media.

For a plasma with electromagnetic oscillations we use the two-fluid description; because of the large difference in electron and ion oscillation frequencies in the solar plasma, the two-time-scale approximation is also relevant. In this case, plasma motion satisfies the following equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e v_e) = 0,$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i v_i) = 0,$$

$$\frac{\partial v_e}{\partial t} + (v_e \nabla) v_e = \frac{e}{m_e} \left( E + \frac{1}{c} v_e \times B \right) - \frac{\nabla P_e}{m_e n_e} + g + \nu_{ei} (v_i - v_e),$$

$$\frac{\partial v_i}{\partial t} + (v_i \nabla) v_i = -\frac{e}{m_i} \left( E + \frac{1}{c} v_e \times B \right) - \frac{\nabla P_i}{m_i n_i} + g - \nu_{ei} (v_e - v_i),$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t},$$

$$\nabla \times B = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4 \pi}{c} \left( e n_e v_e - e n_i v_i \right),$$

$$\nabla \cdot B = 0,$$
Fig. 1a–f. Post-flare loop system of 7 December, 1981, recorded in the emission of the green (5303 Å; FeXIV) and red (6374Å; FeX) coronal lines, showing the development of loop-interaction enhancements: a complex structure recorded in the green-line at 16:36 UT; b at 16:49 UT, an enhancement (A) reaches a maximum brightness in the green-line image; c at the same time, this enhancement is not evident in the red-line image; d at 16:56 UT in the green-line image (A) has faded slightly, while the enhancement (B) has become higher and reached a maximum in brightness; e at the same time, in the red-line image (A) has reached a maximum; by 17:04 UT f, (A) has faded in the red-line image, while (B) has reached a maximum in brightness.

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where $n$ is the number density, $\nu$ the velocity, $P$ the thermal pressure, $E$ the electric field, $B$ the magnetic field, $g$ the gravitational field and $\nu_{ei}$ the collisional frequency between electron and ion. The subscripts $e$ and $i$ indicate the electron and ion components, respectively. Based on the two-time-scale approximation, all the field quantities could be separated into the fast-time-scale and slow-time-scale components:

$$A = (n_e, n_i; \nu_e, \nu_i; P_e, P_i; E, B) = A_f + A_s,$$

and it could be assumed that the ensemble average value of fast time-scale components over the slow-time-scale vanishes:

$$\langle A_f \rangle = 0.$$

On a slow time-scale, a quasi-neutrality condition is valid:

$$\langle e n_e - e n_i \rangle = 0. \quad (3.8)$$

For the hydrogen plasma the result is,

$$n_e^e = n_i^i \equiv n_s. \quad (3.9)$$

Beginning at the electron component of the plasma, from Eq. (3.1),

$$\frac{\partial}{\partial t} n_s + \nabla \cdot \left( n_s \nu_s^e + \langle n_f^s \nu_f^s \rangle \right) = 0, \quad (3.10)$$

and

$$\frac{\partial}{\partial t} n_f^e + \nabla \cdot \left( n_s \nu_f^e + n_f^e \nu_f^e + n_f^i \nu_f^i - \langle n_f^s \nu_f^s \rangle \right) = 0. \quad (3.11)$$

Conversely, from Eq. (3.3) we obtain the lowest order component equation for the fast time-scale, by

$$\frac{\partial n_f^e}{\partial t} \approx \frac{e}{m_e} E_f; \quad (3.12)$$

using Eq. (3.12), one can estimate the terms in Eq. (3.11) as

$$\frac{\nabla \cdot (n_f^e \nu_f^e)}{\partial n_f^e} \sim -\frac{k(n_f^e \nu_f^e)}{\omega n_f^e} \sim \frac{k|e| E_f}{\omega m_e \omega} \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \frac{\omega_{pe}^2}{\omega^2},$$

$$\frac{\nabla \cdot (n_f^e \nu_f^e)}{\partial n_f^e} \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right) \left( \frac{\nu_f^e}{\nu_{Te}} \right),$$

where $\omega_{pe}$ is the electron plasma frequency, and $\nu_{Te}$ the electron thermal velocity:

$$\omega_{pe} = \left( \frac{4\pi n_e e^2}{m_e} \right)^{\frac{1}{2}}, \quad \nu_{Te} = \left( \frac{T_e}{m_e} \right)^{\frac{1}{2}}. \quad (3.13a)$$

where $k_d$ is the Debye wave number and $\tilde{W}_f$ is the wave energy density:

$$k_d = \frac{\omega_{pe}}{v_{Te}}, \quad (3.13b)$$

$$\tilde{W}_f = \frac{E_f^2}{4\pi n_e T_e}, \quad (3.13c)$$

and where $T_e$ is the temperature for electrons (in units of energy). The slow time-scale fluid motion is

$$|\nu_s^e| < \nu_{Te}, \quad (3.14)$$

and the hydrodynamical description for electron motion is valid provided that

$$\nu_{Te}/\nu_\phi \ll 1, \quad (v_\phi \equiv \omega/k) \quad (3.15)$$

where $\omega$ and $k$ are the wave frequency and wave number, respectively. Then Eq. (3.11) could be simplified as

$$\frac{\partial n_f^e}{\partial t} + \nabla \cdot \left( n_s \nu_f^e \right) = 0. \quad (3.16)$$

By combining Eqs. (3.12) and (3.16) we obtain the following estimation values:

$$\nu_f^e \sim \frac{|e| E_f}{m_e \omega} \sim \frac{\omega_{pe}}{\omega} \tilde{W}_f^{\frac{1}{2}} \nu_{Te}, \quad (3.17a)$$

$$\frac{n_f^e}{n_s} \sim \frac{k}{k_d} \left( \frac{\omega_{pe}}{\omega} \right)^2 \tilde{W}_f^{\frac{1}{2}} \ll 1. \quad (3.17b)$$

By using Eqs. (3.17a) and (3.17b) one can compare the last two terms in Eq. (3.10):

$$\frac{n_f^e \nu_f^e}{n_s \nu_s^e} \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \tilde{W}_f^{\frac{1}{2}} \ll 1,$$

For the purposes of this study, we assume that

$$\tilde{W}_f < 1. \quad (3.18)$$

Thus, Eq. (3.10) becomes

$$\frac{\partial}{\partial t} n_s + \nabla \cdot \left( n_s \nu_s^e \right) = 0. \quad (3.19)$$

For the terms corresponding to ions, we may repeat our calculations in a manner similar to the procedure of Eqs. (3.10) through (3.19), resulting in an expression similar to (3.16):

$$\frac{\partial}{\partial t} n_f^i + \nabla \cdot \left( n_s \nu_f^i \right) = 0.$$

Similar to Eqs. (3.17a) and (3.17b), it can be shown that

$$\nu_f^i \sim \left( \frac{\omega_{pe}}{\omega} \right) \tilde{W}_f^{\frac{1}{2}} \nu_{Te} \left( \frac{m_e}{m_i} \right), \quad (3.20a)$$
\[ n_f^i / n_s \sim \left( \frac{k}{k_d} \right) \left( \frac{\omega_{pe}}{\omega} \right)^2 \bar{W}^{1/2} \left( \frac{m_e}{m_i} \right) \ll 1. \] (3.20b)

Thus we also find,
\[ \frac{\partial}{\partial t} n_s + \nabla \cdot (n_s v_s^i) = 0. \] (3.21)

Substituting the corresponding fast and slow components into Eq. (3.3), one obtains the average equation and the fast component equation. It is assumed that the characteristic scale of the slow component is larger than that of the fast component, i.e., \( k_s < k \), and because \( \nu_{ei} \ll \omega_{pe} \), by comparing the terms in the fast component equation, this yields
\[ \frac{\partial}{\partial t} v_f^e \approx \frac{e}{m_e} E_f + \frac{e}{m_e c} [v_f^e \times B_s]. \] (3.22)

From Eq. (3.22), the slow time-scale momentum equation for the electrons is
\[ \frac{\partial}{\partial t} v_s^e + (v_s^e \nabla) v_s^e = \frac{e}{m_e} \left[ E_s + \frac{1}{c} v_s^e \times B_s \right] - \frac{\nabla P_s^e}{m_e n_s} - \nu_{ei} (v_s^e - v_s^i) + g + F_p^e, \] (3.23)

where \( F_p^e \) is the ponderomotive force due to the high frequency oscillations and the MHD interaction:
\[ F_p^e = -\frac{1}{2} \nabla \left( \left( v_f^e \right)^2 \right) + \frac{e}{m_e c} \left( v_f^e \times \nabla \times (\phi_e \times B_s) \right), \] (3.24a)

and
\[ \dot{\phi}_e = \frac{\partial \phi_e}{\partial t} = v_f^e. \] (3.24b)

Similarly, we obtain the slow time-scale momentum equation for ions:
\[ \frac{\partial}{\partial t} v_s^i + (v_s^i \nabla) v_s^i = \frac{e}{m_i} \left[ E_s + \frac{1}{c} v_s^i \times B_s \right] - \frac{\nabla P_i^i}{m_i n_s} + \nu_{ei} (v_s^i - v_s^e) + g + F_p^i, \] (3.25)

where \( F_p^i \) is the ponderomotive force corresponding to the ions:
\[ F_p^i = -\frac{1}{2} \nabla \left( \left( v_f^i \right)^2 \right) - \frac{e}{m_i c} \left( v_f^i \times \nabla \times (\phi_i \times B_i) \right), \] (3.26a)

and
\[ \dot{\phi}_i = \frac{\partial \phi_i}{\partial t} = v_f^i. \] (3.26b)

From Eq. (3.6), the fast time-scale field equation can be derived as
\[ \nabla \times B_f = \frac{1}{c} \frac{\partial E_f}{\partial t} + \frac{4 \pi e}{c} \times \] (3.27)

We may ignore the terms corresponding to ions in Eq. (3.27) in view of Eq. (3.20). Then by use of Eqs. (3.14) and (3.17), Eq. (3.27) becomes
\[ \nabla \times B_f \approx \frac{1}{c} \frac{\partial E_f}{\partial t} + \frac{4 \pi e}{c} n_s v_f^e. \] (3.28)

Hence, by using Eqs. (3.22) and (3.5), we derive the transport equation for fast oscillations from Eq. (3.28):
\[ \nabla \times \nabla \times \phi_e + \frac{1}{c^2} \phi_e + \frac{1}{c^2} \frac{4 \pi e^2}{m_e n_s} \phi_e = - \frac{1}{c^2 \omega_{Be}} \frac{\phi_e}{B_s} \times \left( \frac{B_i}{B_s} \right) - \frac{e}{m_e c} \nabla \times \nabla \times (\phi_e \times B_s) = 0, \] (3.29)

where \( \omega_{Be} = eB_s / m_e c \).

By defining
\[ \rho = n_s (m_i + m_e), \]
we may obtain a set of equations for global coupling MHD equations with ponderomotive force by combining Eqs. (3.19), (3.21), (3.23), (3.25) and (3.29) (Li and Wu 1989):
\[ \frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{U}) = 0, \] (3.30)

\[ \rho \left[ \frac{\partial}{\partial t} \mathbf{U} + (\mathbf{U} \cdot \nabla) \mathbf{U} \right] = \frac{1}{c} \dot{J} \times \mathbf{B} - \nabla P + \rho g + F_p, \] (3.31)

\[ \nabla \cdot \nabla \times (\phi + \frac{1}{c^2} \phi + \frac{1}{c^2} \frac{4 \pi e^2}{m_i m_e} \rho \dot{\phi}) = \frac{\omega_{Be}}{c^2} \frac{\phi}{B_s} \times \left( \frac{B_i}{B_s} \right) + \frac{e}{m_e c} \nabla \times \nabla \times (\phi \times \mathbf{B}) = 0, \] (3.32)

with
\[ F_p = \frac{e}{m_i c} \rho \left( \phi \times \nabla \times (\phi \times \mathbf{B}) \right) - \frac{1}{2} \frac{m_e}{m_i} \rho \nabla \left( \phi \right) \right) \right), \] (3.33a)

\[ \dot{\phi} = \frac{\partial \phi}{\partial t} = v_f^i. \] (3.33b)

The general Ohm’s law is,
\[ \eta \mathbf{J} = \mathbf{E} + (\mathbf{U} \times \mathbf{B}) / c, \] (3.34a)

where the electrical resistivity,
\[ \eta = m_e \nu_{ei} / (n_e c^2), \] (3.34b)

and Maxwell’s equations are,
\[ \nabla \times B = \frac{4\pi}{c} j, \quad (3.35a) \]
\[ \nabla \times E = -\frac{1}{c} \frac{\partial}{\partial t} B, \quad (3.35b) \]
\[ \nabla \cdot B = 0. \quad (3.35c) \]

We have ignored the contribution of the ions in Eq. (3.33a) with the relation \( |F^e_p| \ll |F^i_p| \), and neglected the second order terms for Ohm's law, together with the pressure gradient, gravitational force and ponderomotive force. Because \( \nu_{ei} \ll \omega_{pe} \), the displacement current is dropped in Eq. (3.35a), and the subscript "s" is ignored for electrical and magnetic fields and current.

In the case of \( \omega_{pe} \gg \omega_{Be} \) or \( \partial \parallel B \) (i.e., the high frequency field parallels the magnetic field), then Eqs. (3.32) and (3.33a) are reduced to
\[ \nabla \times \nabla \times f^e_f + \frac{1}{c^2} \nabla \times \left( \frac{4\pi e^2}{c^2 m_e} \rho f^e_f \right) = 0, \quad (3.36) \]
\[ F_p = -\frac{1}{2} \frac{m_e}{m_i} \rho \nabla \left( \langle v_f^e \rangle^2 \right). \quad (3.37) \]

4. Instabilities by solitonic waves in loop coalescence

Apparently the phenomenon of occasional enhancements that appear in the region of loop intersections may be caused by localized loop coalescence and partial magnetic reconnection, with the possibility of an increase in current density, and of heating of the plasma in the immediate vicinity of the x-point. Thus, we present a detailed analysis, including the ponderomotive force, for loop interactions.

We assume that there is a transverse plasmon oscillating along the y-direction, and the unperturbed field is \( B_o \), perpendicular to the x-direction in a slab geometry. Also assuming that the gravitational and compressible effects are negligible (Furth et al. 1963, hereafter referred to as FKR), the relevant equations are:
\[ \frac{\partial}{\partial t} \rho + \nabla \cdot (\rho U) = 0, \quad (4.1) \]
\[ \rho \left[ \frac{\partial}{\partial t} U + (U \cdot \nabla) U \right] = \frac{1}{4\pi} (\nabla \times B) \times B - \nabla P - \frac{1}{2} \frac{m_e}{m_i} \rho \nabla \langle v_f^2 \rangle, \quad (4.2) \]
\[ \frac{\partial}{\partial t} B = \nabla \times (U \times B) + \frac{c^2}{4\pi} \eta \nabla^2 B, \quad (4.3) \]
\[ \nabla \times \nabla \times f_f + \frac{1}{c^2} \frac{\partial^2 f_f}{\partial t^2} + \frac{1}{4\pi} \frac{\rho}{c^2 m_e m_i} v_f^2 = 0, \quad (4.4) \]
\[ \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}, \quad (4.5) \]

Examining the unperturbed state of Eqs. (4.1) through (4.7), the values are:
\[ U_o = 0, \quad \rho_o = \tilde{\rho} + \rho_0 \langle x \rangle, \quad (4.8) \]
\[ B_o = B_{oy} (x) \tilde{y} = B \tanh \left( \frac{x}{L_s} \right) \tilde{y}. \quad (4.9) \]

Within the current sheet (i.e., \( |\tilde{x}| = |x/L_s| \ll 1 \)), the magnetic field can be approximated by
\[ B_o = \tilde{B} \frac{x}{L_s} \tilde{y}, \quad (4.10) \]
where \( L_s \) represents the characteristic length of the current sheet. From Eq. (4.2) the result is
\[ \rho_0 \langle x \rangle \approx \frac{1}{2} \tilde{\rho} \frac{m_e}{m_i} \langle v_{fo}^2 \rangle, \quad (4.11) \]
in the case of
\[ \frac{1}{8\pi} \tilde{B}^2 \left( \frac{x}{L_s} \right)^2 \ll \frac{1}{2} \frac{m_e}{m_i} \langle v_{fo}^2 \rangle, \quad (4.12) \]
where \( c_s \) is the sound speed. The fast oscillation velocity of the electron can be expressed by
\[ v_{fo} = \frac{1}{2} \left[ v_o (r,t) e^{i\omega_0 t} + c.c. \right], \quad (4.13) \]
with \( \omega_0 \) being the fast oscillation frequency of a transverse plasmon,
\[ \omega_0^2 \approx \frac{4\pi e^2}{m_e m_i} \rho, \quad (4.14) \]
and c.c. representing complex conjugation of the first term. In this case Eq. (4.4) becomes
\[ i \frac{\partial}{\partial t} v_o + \frac{c^2}{2m_e} \nabla^2 v_o + \frac{\omega_{pe} m_e}{8c_s^2 m_i} |v_e|^2 v_o = 0, \quad (4.15) \]
with
\[ \left| \frac{1}{\omega_0} \frac{\partial}{\partial t} \ln v_o \right| \ll 1. \quad (4.16) \]

For the transverse plasmon, \( v_o = v_o (x,t) \tilde{y} \), and Eq. (4.15) can be written as a standard nonlinear Schrödinger equation:
\[ i \frac{\partial v_o}{\partial t} = -\frac{1}{2} \frac{\partial^2}{\partial x^2} v_o - \beta |v_o|^2 v_o \quad (4.17) \]
with
\[ \xi = \frac{1}{c} \left( \omega_{pe} \right)^{\frac{1}{3}} x, \quad \beta = \frac{\omega_{pe}}{8c_s^2} \mu, \quad \mu = \frac{m_e}{m_i}, \quad (4.18) \]

From Eq. (4.17) it is clearly indicated that the waves can be considered as a wave packet and its wave function is proportional to \( \psi \); the packet can produce a self-generated potential \( \beta |v_o|^2 \). Since \( \beta \) is positive, this self-generated potential has attractive characteristics. In other words, the nonlinear wave packet can be self-centered and forms a stable structure called a "solitary wave". For the steady state, the solution of Eq. (4.17) in the form of a solitary wave is (Li 1987):

\[ \psi = \psi_0 \text{sech} \left( \sqrt{\beta \psi_0^2} \xi \right) e^{i\phi}, \quad (4.19a) \]

\[ \phi = \frac{t}{2} \beta \left( \psi_0^2 \right)^2 + \phi_o. \quad (4.19b) \]

Thus Eq. (4.11) becomes

\[ \rho_0'(x) = -\frac{1}{c_s^2 \sqrt{\mu}} \mu \left( \psi_0^2 \right)^2 \text{sech}^2 \left( \frac{x}{\epsilon_o} \right), \quad (4.20) \]

where \( \epsilon_o \) is the width of the solitary wave:

\[ \epsilon_o = \frac{\sqrt{8}}{\mu} \left( \frac{c_s}{\omega_{pe}} \right) \left( \frac{c_s}{v_0^2} \right) \quad (4.21) \]

The width of a current sheet can be estimated from Eq. (4.12), such as

\[ \nabla x = \epsilon \ll \sqrt{\mu} \left( \frac{\psi_0}{v_0} \right) L_s, \quad (4.22) \]

with \( v_A \) being the Alfven velocity. A current sheet with antiparallel magnetic field lines is unstable to breaking and relinking of field lines, in which the enhanced radio emission is occurring (Priest 1986); and the plasma is expelled from the region of emission accumulation by the ponderomotive force, resulting in a localized drop in density, as shown by Eq. (4.20).

Examining the perturbed state, by using Eqs. (4.1) through (4.3), we have

\[ \frac{\partial U}{\partial t} = -\nabla P + \frac{1}{4\pi} \left( \nabla \times B \right) \times B - \frac{1}{2} \mu \rho \nabla |v_o|^2, \quad (4.23) \]

\[ \frac{\partial B}{\partial t} = \nabla \times (U \times B_o) - \frac{1}{4\pi} \eta c^2 \nabla \times (\nabla \times B), \quad (4.24) \]

\[ \frac{\partial \rho}{\partial t} = -(U \cdot \nabla) \rho_o, \quad (4.25) \]

\[ \nabla \cdot U = 0, \quad \nabla \cdot B = 0. \quad (4.26) \]

Taking all of the perturbed quantities, such as

\[ A(x, y, t) = A(x) e^{iky} e^{\gamma t}, \quad (4.27) \]

then from Eqs. (4.23) through (4.26) we obtain

\[ \psi'' = \alpha^2 \psi \left( 1 + \frac{\gamma_{TR}}{\alpha^2} \right) - i k \tau_R U_x F, \quad (4.28) \]

\[ \left( \phi_o U_x \right)' = \alpha^2 U_x \left[ \phi_o + \frac{s^2}{\gamma_{TR}} G_o + \frac{S^2 F^2}{\gamma_{TR}} \right] \]

\[ + \left( \frac{i}{k \tau_R} \right) \psi \alpha^2 S \left( F - \frac{F''}{\gamma_{TR}} \right), \quad (4.29) \]

with

\[ B_{ov}(x) = BF(x), \quad \psi = B_x / B, \quad \rho_o = \rho \phi_o (x), \quad (4.30a) \]

\[ G_o = \left( -\frac{1}{4} \mu \frac{d}{dx} |v_o|^2 \right) \frac{1}{\rho} \left( \frac{d}{dx} \rho_o \right) \frac{\gamma_{H}}{\tau_R}, \quad (4.30b) \]

\[ x = L_s \tilde{x}, \quad \alpha = k L_s, \quad S = \tau_R / \tau_H, \quad (4.30c) \]

\[ \tau_R = 4\pi L_s^2 / \eta c^2, \quad \tau_H = 4\pi L_s \sqrt{\mu} / B; \quad (4.30d) \]

where the superscript prime "'" represents the derivative with respect to \( \tilde{x} \). In the absence of electromagnetic emission, i.e., \( G_o = 0 \), Eqs. (4.28) and (4.29) are reduced to those discussed by FKR.

The existence of the current sheet divides the conducting plasma medium into two regions: the region outside the current sheet with infinite conductivity (\( \eta \to 0 \)) and the region inside the current sheet with finite resistivity. Concerning the transverse plasma waves, \( G_o \) is negligible outside the current sheet, because the plasmon appears in the form of a solitary wave inside the sheet. Hence one obtains the jump condition across the sheet in the outer region:

\[ \Delta \equiv (\psi' - \psi'') / \psi(0) = 2 \left( \alpha^{-1} - \alpha \right). \quad (4.31) \]

The magnetic field lines, driven by a Lorentz force, are diffused towards the dissipation region from both sides, and the field configuration is distorted; then the field function, \( \psi \), may be expanded as a series of a small parameter

\[ \delta \sim |\tilde{x}| \ll 1, \quad (4.32) \]

i.e.,

\[ \psi = \psi_0 + \psi_1 + \psi_2 + \cdots, \quad (4.33) \]

where \( \psi_m \) is proportional to \( \delta^m \).

In a similar manner to that given by FKR, under a constant \( \psi_0 \) approximation, the jump condition within the current sheet can be expressed as

\[ \Delta_l = 4\pi \left( \frac{\gamma_{TR}}{\alpha S} \right)^{5/4} \Gamma \left( \frac{3}{4} \right) \left( 1 + \zeta \right)^{1/4} \quad (4.34) \]

\[ \zeta = \frac{4\pi \left( \frac{\gamma_{TR}}{\alpha S} \right)^{5/4} \Gamma \left( \frac{3}{4} \right)}{\left( 1 + \zeta \right)^{1/4}} \]
with $\Gamma$ being the gamma function and

$$\zeta = d^2_{o}/(\gamma \tau_R), \quad (4.35a)$$

$$d^2_{o} = \pi \mu^2 \left( \frac{v^0_o}{c_s} \right)^2 \left( \frac{v^0_o}{v_A} \right)^2 \left( \frac{L_s}{\epsilon_o} \right)^4, \quad (4.35b)$$

$$v_A = \frac{\vec{B}}{\sqrt{4\pi \rho}}. \quad (4.35c)$$

By equating the jump conditions inside and outside the current sheet, the dispersion relation for the growth rate $\gamma$ from Eqs. (4.31) and (4.34) is

$$4\pi (\gamma \tau_R)^{5/4} \frac{\Gamma(1/2)}{\Gamma(1/4)} (1 + \zeta)^{1/4} = 2 \left( \frac{1}{\alpha} - \alpha \right). \quad (4.36)$$

This is the key equation for determining the resistive instability by solitary waves ($\zeta \gg 1$). The characteristic time-scale with the magnetic energy being converted into heat and fast particle energy is determined by the growth rate $\gamma$ of Eq. (4.36). In this case the power density for magnetic energy dissipation may be estimated as follows,

$$Q = \frac{\psi^2}{\sigma} = \frac{1}{16\pi^2 \sigma} \left( \epsilon \text{rot} B \right)^2 \approx \frac{c^2}{4\pi^2 \sigma} \frac{B^2}{d^2}, \quad (4.37)$$

where the electrical conductivity $\sigma = 1/\eta$, $\text{rot} B \sim 2B/d$, and $d$ is the thickness of a current sheet.

When the reconnection process occurs, the constant $\psi_0$ approximation is no longer valid; thus we need to examine this localized instability. By introducing the Fourier transformation such as

$$\psi = \int_{-\infty}^{\infty} \hat{\psi}(\theta) e^{-i\theta \phi} d\theta, \quad (4.38a)$$

$$U = \int_{-\infty}^{\infty} \hat{U}(\theta) e^{-i\theta \phi} d\theta, \quad (4.38b)$$

corresponding to the inner region derived from Eqs. (4.28) and (4.29),

$$\frac{s^2}{\gamma \tau_R} \left[ \frac{d^2 \hat{U}(\theta)}{d\theta^2} + \frac{d}{d\theta} \left( \frac{\theta^2 + \alpha^2}{\theta^2 + (\gamma \tau_R)} \frac{d \hat{U}(\theta)}{d\theta} \right) \right] - \hat{U}(\theta) \left( \frac{\theta^2}{\alpha^2} + 1 \right) = 0. \quad (4.39)$$

It is noted that Eq. (4.39) is identical to the FKR Eq. (E5) by setting $\zeta = 0$. It is necessary to seek such an eigenvalue of growth rate $\gamma$, so that the function $U$ (or $\psi$), defined by Eq. (4.38) satisfies the following boundary condition:

$$U \to 0, \quad \text{as } \vec{x} \to \vec{x}_b, \quad (4.40)$$

where $\vec{x}_b$ is the boundary of the inner region. Eq. (4.39) yields the eigenvalue of $\gamma_b$ to be (Li & Wu 1989; Li 1990):

$$\gamma_b \approx \frac{\partial \gamma}{\partial \nu}, \quad (4.41a)$$

with

$$d_{eff} = \left( \frac{\sqrt{4\pi} \ L_s}{d_0 \ L_y} \right) \frac{L_s}{d_0} \alpha, \quad (4.41b)$$

and $L_y$ is the characteristic scale of the current sheet along the $y$-direction. This instability, determined by Eq. (4.41), is responsible for carrying away the products of field annihilation at a fast enough rate.

5. Computational results

From Eq. (4.36) we can see that the growth rate of the instability is $\gamma \sim \eta^{1/2}$, and its value is

$$\gamma \approx \frac{1}{d_0^2} \frac{\alpha^2}{2\pi} \frac{(1 - \alpha)}{\Gamma(1/4) \Gamma(1/4)} \frac{1}{\tau_R} \quad (sec^{-1}) \quad (5.1)$$

For the post-flare coronal loop system in the green line, the temperature and electron density in the region of the current sheet are $T_e = 2 \times 10^6 K$, $n_e = 5 \times 10^{10} cm^{-3}$ (Smatt & Zhang 1990; Zhang & Smartt 1991; Smartt et al. 1993). If we use $\vec{B} = 100G$ (Lang & Willson 1984) $v^0_o = 2.2 \times 10^7 cm/sec$ and $L_s = 4 \times 10^5 cm$, we obtain $c_s = 2.2 \times 10^7 cm/sec$, $v_A = 9.9 \times 10^7 cm/sec$ and $\omega_{pe} = 1.3 \times 10^{10} sec^{-1}$ [$\eta = 6 \times 10^{-17}$ esu (Allen 1973)]. By using Eqs. (4.30c), (4.30d), (4.20) and (4.35b), the result is $\epsilon_o = 2.8 \times 10^2 cm$, $d_o = 4.3 \times 10^2 cm$, $\tau_R = 3.7 \times 10^7 (sec)$, $\gamma = 1.4 \times 10^2 (sec)$ and $S = 2.6 \times 10^9$. Thus if we use $\alpha = 10^{-3}$, from Eq. (5.1) the result is

$$\gamma = 9.8 \times 10^{-4} \quad (sec^{-1})$$

and the corresponding characteristic time-scale is

$$t = \frac{1}{\gamma} \approx 17 \text{ minutes},$$

which relates to the lifetime of 20-30 minutes for the ninety events as discussed in Sect. 2.

Furthermore, on the basis of Eq. (4.37) of the power density for magnetic energy dissipation, the estimated energy, $E$, released from the interaction region of the loops is as follows:

$$E \approx \frac{c^2 \eta}{4\pi^2} B^2 (Ad \tau) \approx \frac{c^2 \eta}{4\pi^2} \frac{B^2}{d} A t, \quad (5.2)$$

where $A$ is the area of interaction region. Using the values $d = 4 cm$, $A = 4 \times 10^{18} cm^2$, $B = 100G$, and by the above values of $t$ (= 17 minutes) and $\eta$, we derive

$$E \approx 1.4 \times 10^{38} \text{ ergs},$$

which is the same order of magnitude as those estimated in Zhang & Smartt (1991) and Smartt et al. (1993).
6. Conclusion

During the coalescence of two coronal loops, the plasma and magnetic flux may be compressed towards the current sheet from both sides driven by a Lorentz force, leading to a resistive instability. The magnetic field is annihilated and magnetic energy converted into kinetic energy of particles, thermal energy of the plasma and radiation by Ohmic dissipation, followed by enhanced electromagnetic emission occurring in the current sheet (Priest 1986). This high-frequency emission can be self-centered and forms an electromagnetic solitary wave caused by a modulation instability. In this case the characteristic timescale for magnetic energy release is determined by Eq. (4.36). Additionally, the solitary waves can also result in a localized instability by the coupling effect of the ponderomotive force. The hot plasma, as a product of magnetic field dissipation, is quickly removed due to this instability. The characteristic time-scale, $t_p$, of this process is determined by Eq. (4.41a), and is estimated to be $\approx 3 \times 10^{-8}$ sec. Thus the thickness of the dissipation layer can remain at a very small value, thereby allowing sufficient power in the magnetic field dissipation.

The basic consistency of our computation results with the observational data indicates that the phenomenon of occasional enhancements that appear at the site of loop interactions can be interpreted by this theory of magnetic field reconnection driven by a ponderomotive force.

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