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Coupling Alfvénic and Ion-Acoustic Solitons

WU De-jin, WANG De-yu
Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008

Carl-Gunne Fälthammar
Department of Plasma Physics, Royal Institute of Technology, Stockholm, Sweden

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An exact nonlinear equation governing the coupling Alfvénic and ion-acoustic solitons and criterion for their existence are presented. For the case of low-β plasmas, the two modes decouple. In the small amplitude limit, the analytical results lead to KdV solitons.

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It is well known that the ideal magnetohydrodynamics (MHD) Alfvén wave does not have dispersion at a low frequency (\(\ll \)plasma characteristic frequencies) and a long wavelength (\(\gg \)plasma characteristic lengths). However, when the perpendicular wavelength, \(2\pi/k_{\perp}\), becomes comparable to the ion gyroradius, \(\rho_i\), ions can no longer follow the magnetic lines of force, whereas electrons are still attached to the field because of their small Larmor radius, thus a charge separation can be produced due to the finite Larmor radius effect. This causes the Alfvénic wave not only to have a dispersion for an oblique propagation, but also to couple to the ion-acoustic wave.\(^1\)

The above dispersive Alfvén wave, called “kinetic Alfvén wave (KAW here after)”, combined with the nonlinear steepening, may lead to the formation of solitons called the “solitary kinetic Alfvén waves (SKAWs)”. The SKAWs have been investigated by many authors. Hasegawa et al.\(^2\) and Yu et al.\(^3\) investigated the existence of SKAWs propagating in an oblique direction with respect to the ambient magnetic field in a plasma with \(\alpha \equiv (\beta /2)/Q \gg 1\), where \(\beta\) and \(Q\) are the ratio of thermal pressure (by electrons) to magnetic pressure (by the ambient magnetic field) and the ratio of electron mass to ion mass respectively. They showed the existence of SKAWs accompanied with a hump density soliton, and the KdV soliton corresponding to the small amplitude limit. A similar case, but for \(\alpha \ll 1\), was considered by Shukla et al.,\(^4\) Kalita et al.\(^5\) and Wu et al.\(^6\) They showed the existence of SKAWs but accompanied with a dip density soliton, and gave the KdV soliton as the small amplitude limit. Also, Wu et al.\(^6,7\) obtained at the first time an analytical solution of the finite-amplitude SKAWs and applied it to the earth auroral observations by the Freja satellite. A more general case of arbitrary \(\alpha\) value was investigated by Wu et al.\(^8\) They showed that both the SKAWs accompanied with dip and hump density solitons can exist in a plasma with \(\alpha \sim 1\).

The above mentioned authors all investigated only Alfvénic solitons caused by the finite Larmor radius effect. Therefore it is of interest to study both the Alfvénic and ion-acoustic solitons and their coupling caused by the finite Larmor radius effect. Recent, using the perturbation expansion method, Ghosh et al.\(^9\) investigated this problem in the small amplitude limit for the case of \(\alpha \gg 1\), and derived KdV equations governing small amplitude Alfvénic and ion-acoustic solitons. In the present letter, we derive an exact nonlinear equation governing the finite amplitude coupling Alfvénic and ion-acoustic solitons and criteria for the existence

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of these solitons. Also, we show that the two modes decouple in the case of low-\(\beta\), and lead to the KdV solitons in the small amplitude limit.

Consider a homogeneous plasma in a uniform ambient magnetic field \(B_0\) along the \(z\) direction. For one-dimensional plane waves oblique propagation with the wave vector \((k_x, 0, k_z)\), we assume further the wave to be stationary in the moving frame defined by

\[
\eta = k_x x + k_z z - \omega t,
\]

where the space and time are normalized by \(v_A/\omega_{ci}\) and \(\omega_{ci}^{-1}\) respectively, \(v_A\) is the Alfvén velocity and \(\omega_{ci}\) is the ion gyrofrequency. In this one-dimensional stationary frame, the equations governing the dynamics of the wave in a cold ion \((T_i \ll T_e)\) plasma are as follows:\(^6\)

\[
\begin{align*}
&d_\eta [n_e (k_z v_{ez} - \omega)] = 0, \\
&d_\eta [n_i (k_z v_{iz} + k_x v_{ix} - \omega)] = 0, \\
&(\omega - k_z v_{ez})d_\eta v_{ez} = \alpha(E_z + k_z d_\eta \ln n_e), \\
&(k_x v_{ix} + k_z v_{iz} - \omega)d_\eta v_{iz} = Q\alpha E_z, \\
&v_{ix} = -Q\omega d_\eta E_x, \\
&k_z d_\eta B_y = -n_i v_{ix}, \\
&Q\alpha d_\eta (k_z E_z - k_x E_x) = \omega d_\eta B_y, \\
&n_i = n_e = n,
\end{align*}
\]

where the density, velocity, perturbed electric and magnetic fields are normalized by \(n_0, v_A, T_e, \omega_{ci}/(e v_A)\) and \(B_0\) respectively, \(n_0\) and \(T_e\) are the unperturbed plasma density and the electric temperature in the unit of eV respectively, \(e\) is the basic charge, \(d_\eta\) denotes derivative with respect to \(\eta\), and Eq. (9) indicates the charge neutrality assumption.

From Eqs. (2)-(9), it is easy to find that the linear dispersion equation is

\[
(1 + Q + Qk_z^2)M_z^4 - (1 + Q + Q\alpha k_z^2)vM_z^2 + Q\alpha = 0.
\]

It represents the coupling of Alfvénic and ion-acoustic waves,\(^10\) and its two roots are

\[
M_z^2 = \frac{1 + Q\alpha k_z^2}{2} \left\{ 1 \pm \left[ 1 - 4Q\alpha \left( \frac{1 + Qk_z^2}{1 + Q\alpha k_z^2} \right)^2 \right]^{1/2} \right\},
\]

where \(M_z = \omega/k_z\), and a trivial \(Q\) term in \(1 + Q\) has been neglected. In a low-\(\beta\) case, namely \(\beta/2 = Q\alpha \ll 1\), the two modes decouple, and the corresponding dispersion relations for Alfvénic and ion-acoustic modes, respectively, are

\[
M_{zA}^2 = \frac{1 + Q\alpha k_z^2}{1 + Qk_z^2}, \quad M_{zs}^2 = Q\alpha \frac{1 + Qk_z^2}{1 + Q\alpha k_z^2}.
\]

Integrating Eqs. (2)-(9), by use of the localized boundary conditions for the solution of solitary waves, \(n = 1, v_{ez} = v_{iz} = d_\eta = 0\) for \(|\eta| \to \infty\), we can obtain that the nonlinear equation governing the coupling solitary wave solutions is as follows:

\[
(d_\eta n)^2 + 2K(n; M_z, k_z) = 0,
\]
where the Sagdeev potential \( K(n; M_z, k_z) \) is

\[
K(n; M_z, k_z) = -\frac{(1 + Q)}{Q M_x^2 k_z^2 (1 - \alpha n^2/M_x^2)^2} \left[ \Phi_1 + \frac{\alpha}{M_x^2} \Phi_2 + Q \frac{\alpha}{M_x^2} \left( \Phi_3 + \frac{\alpha}{M_x^2} \Phi_4 \right) \right]
\]  
(14)

with

\[
\Phi_1 = \frac{1}{6} (n - 1)^2 \left( 3 M_x^2 - 1 - \frac{2}{n} \right),
\]
(15)

\[
\Phi_2 = -n(n - 1)(M_x^2 n + 1) + (M_x^2 + 1)n^2 \ln n,
\]
(16)

\[
\Phi_3 = -\frac{n^2 - 1}{2} + (M_x^2 + 1)(n - 1)n - M_x^2 n^2 \ln n,
\]
(17)

\[
\Phi_4 = \frac{1}{2} M_x^2 n^2 (n^2 - 1) - (1 + M_x^2)(n - 1)n^2 + n^2 \ln n.
\]
(18)

The root of the Sagdeev potential \( K(n; M_z, k_z) \), assumed to be the nearest one by \( n = 1 \),

\[
n_m = n_m(M_z, \alpha),
\]
(19)
determines the maximum variation of plasma density in SKAWs. Neglecting the terms proportional to \( Q \), namely \( \Phi_3 \) and \( \Phi_4 \), in Eq. (14), it is easy to find that, in the limits of \( \alpha \gg 1 \) and \( \alpha' \ll 1 \), the Sagdeev potential of Eq. (14) are, respectively, the same as that obtained by Hasegawa et al.\(^2\) and Shukla et al.\(^4\).

From Eqs. (13)-(19), one can obtain the conditions for existence of solitary wave solutions as follows:\(^3\)

\[
(1 - M_x^2) \left( 1 - \frac{\alpha}{M_x^2} \right) \left( 1 - Q \frac{\alpha}{M_x^2} \right) < 0,
\]
(20)

\[
(1 - M_x^2 n_m) \left( 1 - \frac{\alpha n_m^2}{M_x^2} \right) \left( 1 - Q \frac{\alpha n_m}{M_x^2} \right) > 0,
\]
(21)

\[
\left( 1 - \frac{\alpha}{M_x^2} \right) \left( 1 - \frac{\alpha n_m^2}{M_x^2} \right) > 0.
\]
(22)

The above inequalities (20)-(22) determine ranges of the parameters \((\alpha, n_m \) and \( M_x \)) for the existence of solitary wave solutions of the coupling mode, hence, give the criteria for existence of solitons.

In a low-\( \beta \) plasma, the two modes decouple. For the Alfvénic mode of \( M_{zA}^2 \sim 1 \) and the ion-acoustic mode of \( M_{zA}^2 \sim \beta/2 \equiv Q \alpha \ll 1 \), the criteria for existence of solitons become, respectively, as follows:

\[
(1 - M_{zA}^2) \left( 1 - \frac{\alpha}{M_{zA}^2} \right) < 0, \quad (1 - M_{zA}^2 n_m) \left( 1 - \frac{\alpha n_m^2}{M_{zA}^2} \right) > 0,
\]
(23)

\[
1 - Q \frac{\alpha}{M_{zA}^2} > 0, \quad 1 - Q \frac{\alpha n_m}{M_{zA}^2} < 0.
\]
(24)

Let \( N = n - 1 \) represents the deviation of the perturbed density \( n \) from the ambient density \( n_0 = 1 \), the small amplitude limit means

\[
|N| \leq |N_m| \ll 1,
\]
(25)
where \( N_m = n_m - 1 \) is the amplitude of density solitons.

For the small amplitude Alfvénic and ion-acoustic modes, assuming \( \Delta_A \equiv M_{eA}^2 - 1 \) and \( \Delta_s \equiv M_{es}^2/(Q\alpha) - 1 \) to be the same orders as \( N_m \), one can easily find that the Sagdeev potential \( K(n; M_s, k_z) \) of Eq. (14) becomes, respectively, at the lowest order,

\[
K_A(N; \Delta_A, k_z) = -\frac{11 + Q}{3} \frac{1 - Q\alpha}{Qk_z^2} \left( N + \frac{3}{2} \Delta_A \right) N^2,
\]

\[
K_s(N; \Delta_s, k_z) = \frac{11 - Q^2}{3} \frac{1 - Q\alpha}{Qk_z^2} \left( N - \frac{3}{2} \Delta_s \right) N^2.
\]

According to Eqs. (19), (23) and (24), the amplitudes of density perturbation of Alfvénic and ion-acoustic modes are respectively,

\[
N_{mA} = -\frac{3}{2} \Delta_A, \quad N_{mA}(1 - \alpha) < 0,
\]

\[
N_{ms} = \frac{3}{2} \Delta_s, \quad N_{ms} > 0.
\]

And the nonlinear Eq. (13) governing solitons, in the small amplitude limit, may be written as follows:

\[
(d_n N)^2 = 4 \frac{N^2}{D_{A(s)}^2} \left( 1 - \frac{N}{N_{mA(s)}} \right),
\]

It is the standard form of the KdV equation in the travelling-wave frame, and has a soliton solution as follows:

\[
N = N_{mA(s)} \text{sech}^2 \left( \frac{\eta}{D_{A(s)}} \right),
\]

where \( D_{A(s)} \) character the width of Alfvénic and ion-acoustic solitons, they are

\[
D_A = \left[ -6 \frac{Qk_z^2}{1 + Q} \frac{1 - \alpha}{(1 - Q\alpha)N_{mA}} \right]^{1/2},
\]

\[
D_s = \left[ 6 \frac{Qk_z^2}{(1 - Q^2)(1 - Q\alpha)} \frac{\alpha}{N_{ms}} \right]^{1/2}.
\]

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