A self-modulation model of solar radio emission†

WANG De-yu1 HUANG Guang-li1 MAO Ding-yi2
1Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008
2Department of Mathematics and Physics, Hehai University, Nanjing

Abstract We considered the possibility of self-modulation instabilities in waves propagating in the solar corona. We found that such instabilities (both longitudinal and transverse) can occur in decimeter and meter radio bursts but not in optical or X-ray emissions. The model is consistent with the observed fine structures in solar meter wave bursts.

Key words: solar radio emission—nonlinear process—wave modulation

1. INTRODUCTION

In the study of the fine structure in solar radio bursts, it is usual to consider some kind of oscillation occurring in the magnetic tubes (see Aschwanden’s review article[1]). Models of this type can well explain the sub-second periodic structures with equal[2] or unequal intervals[3]. Using a three-wave resonance interaction, Wang De-yu et al.[4] interpreted the solar spike emission with ms structures[5] and the nonlinear modulation process in short cm radio bursts[6,7]. Huang Guang-li et al.[8], using plasma instability induced enhancement in radio emission, interpreted the fine structure superimposed on the continuum. These processes have contributed to our understanding of the mechanism of the fine structures.

In another direction, space satellites have already detected many cases of modulation of whistler and low-frequency mixed waves. These cases are generally regarded as nonlinear effects of waves propagating in a plasma[9,10], or as beats produced by neighbouring frequencies[11]. Similar effects occur when waves propagate in the solar coronal plasma. The aim of this paper is to study the manifestations of this self-modulation process in solar radio bursts. Differing from space measurement, solar radio observation is made far from the source: we can observe only the flux of the waves (optical and X-ray) but we can not measure directly the electric field.

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This paper is organized as follows: Section 2 gives the nonlinear equation of wave propagation under the geometric optics approximation, Section 3 analyses the condition for self-modulation instability to occur and Section 4 discusses the specific case of the solar coronal plasma.

2. EQUATION OF NONLINEAR PROPAGATION OF WAVES IN PLASMA

Let wave $(\omega, k)$ propagate in a magnetized, homogeneous plasma. Because of nonlinear effects, the dispersion equation depends not only on the wave number $k$, but also on the wave amplitude $|\Psi|^2$. The electric vector of the wave is

$$E = 1/2 \{ \Psi(x, z, t) \exp[i(kz - \omega t)] + c.c. \}, \quad (1)$$

here, $\Psi(x, z, t)$ is a slowly varying amplitude which, under the geometric optics approximation\[^{[12]}\] satisfies the equation,

$$\left( \frac{\partial}{\partial t} + V_s \frac{\partial}{\partial z} \right) \Psi + \frac{1}{2} V_s' \frac{\partial^2}{\partial z^2} \Psi + \frac{1}{2} T \frac{\partial^2}{\partial x^2} \Psi - \Delta \omega \Psi = 0, \quad (2)$$

where $V_g = \partial \omega / \partial k$ is the group velocity of the wave, $V_g' = \partial V_g / \partial k = \partial^2 \omega / \partial k^2$ is the dispersion of the group velocity, $T = (\partial^2 \omega / \partial k^2)_{k \rightarrow 0}$ is the dispersion in the direction perpendicular to the propagating and $\Delta \omega$ is the nonlinear frequency drift. We now consider electromagnetic waves propagating along the magnetic field (the $z$-axis). Under the cold plasma approximation and the condition $\omega_{pe} < \omega_{ce}$, the linear dispersion equation can be written as

$$\chi^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{R}{u(u - 1)}, \quad (3)$$

where $R = (\omega_{pe}/\omega_{ce})^2, u = \omega/\omega_{ce}, \omega_{pe}$ being the electron plasma frequency and $\omega_{ce}$, the electron cyclotron frequency. Under these conditions the solution is

$$V_s = \frac{2cN}{2 + \frac{R}{u(u - 1)^2}}, \quad (4a)$$

$$V_s' = \left\{1 - \left( \frac{V_s}{C} \right)^2 \left[1 - \frac{R}{(u - 1)^3} \right] \right\} \left( \frac{V_s}{k} \right), \quad (4b)$$

$$T = \left\{1 - \frac{R}{2(u^2 - R)(u - 1)} \right\} \left( \frac{V_s}{k} \right), \quad (4c)$$

$$\Delta \omega = - \left( \frac{kV_s}{2} \right) \left[ \delta n - d_z \left( b_z + \frac{k}{\omega} V_s \right) \right], \quad (4d)$$

where $\delta n, b_z, V_s$ are the variations in the particle density, magnetic field and velocity field
under the low-frequency perturbation, and

\[ d_1 = \frac{N^2 - 1}{N^2} = \frac{R}{R - u(u - 1)}, \]

\[ d_2 = \frac{\partial[\omega^2(N^2 - 1)]}{N^2 \omega \partial \omega} = \frac{R}{(u - 1)[u(u - 1) - R]} . \]  

Let now the perturbing low-frequency wave be \((\Omega, K)\). From the linear MHD equations for low-frequency waves and taking into account the ponderomotive force of high-frequency waves on low-frequency waves, we have, in the coordinate system \(s = K_x x + K_z z - \Omega t\),

\[ \delta n = \frac{N^2 |p|^2}{16 \pi \rho_0} \left[ \frac{k_x^2 (d_2 - d_1)}{L} - \frac{K_z}{H} \left( d_1 K_z - \frac{K d_2}{\omega} \Omega \right) \right], \]

\[ b_z = \frac{N^2 |p|^2}{16 \pi \rho_0} \frac{k_z^2}{H} d_2, \]

\[ V_z = - \frac{N^2 |p|^2}{16 \pi \rho_0} \frac{\Omega}{H} \left( d_1 k_z - \frac{k}{\omega} d_z \Omega \right), \]

where

\[ d_1 = \frac{\partial[\omega(N^2 - 1)]}{N^2 \partial \omega} = d_2 - d_1, \]

\[ L = \Omega^2 - (V_A^2 + c_s^2) k_x^2 - V_A^2 K_z^2, \]

\[ H = \Omega^2 - c_s^2 k_z^2, \]

\(V_A\), being the Alfvén wave velocity and \(C_s\), the sound velocity. Substituting (6) and (7) in (4d) gives the expression for the nonlinear frequency drift under the action of the low-frequency wave:

\[ \Delta \omega = \left( \frac{kV_A N^2}{32 \pi \rho_0} \right) |p|^2 \frac{1}{LH} \left\{ d_2^2 k_x^2 H + L \left( d_1 K_z - \frac{k \Omega}{\omega} d_2 \right)^2 \right\} . \]

Thus, we see that the nonlinear frequency drift is proportional to the square of the amplitude of the propagating wave. Hence, (2) is a nonlinear parabolic type equation, sometimes called a nonlinear Schrödinger equation. We shall now ask, under what conditions is this equation unstable against the propagation induced low-frequency perturbation, or in other words, can the so-called self-modulation instability take place.

### 3. ANALYSIS OF SELF-MODULATION INSTABILITY

We use the method of small perturbation \[^1\] to address the question of self-modulation
Let
\[ \delta \Psi = \delta \Psi_0 \exp \left[ -i(\Delta \omega)_s t + \phi(s) \right] \]
where \( \delta \Psi \ll \Psi_0, \phi(s) \ll (\Delta \omega)_s t \). Substituting (9) in (12) and referring to the \( s \) coordinate system, we obtain
\[
\left[ (\Omega - K_z^2) - \frac{1}{4} \left( V_s^2 + T K_z^2 \right) \right] \frac{L H}{G(\Omega, K)} (V'_s K_z^2 + T K_x^2) |\Psi_0|^2 = \left( \frac{k V_s N^2}{32 \pi \rho_0} \right)^2 \left\{ d_1^2 - \frac{k \Omega}{\nu} \right\} |\Psi_0|^2.
\]
where
\[
G(\Omega, K) = \left( \frac{k V_s N^2}{32 \pi \rho_0} \right)^2 \left\{ d_1^2 - \frac{k \Omega}{\nu} \right\} |\Psi_0|^2.
\]

We now consider the necessary conditions for instability on the basis of (10) for three cases separately.

3.1 \( K_z = 0 \). This corresponds to a purely longitudinal perturbation. From (10) we get
\[
(\Omega - K_z V_s^2)^2 = \frac{1}{4} V_s^2 K_z^2 + \left( \frac{k V_s N^2}{32 \pi \rho_0} \right) \left( \frac{1}{H} \left( d_1 K_z^2 - \frac{k \Omega}{\nu} \right) V_s^2 K_z^2 \right) |\Psi_0|^2.
\]
From (4b) we know that, for \( u > 1 \), we have \( V'_s > 0 \). Hence instability can occur only for \( H < 0 \) or \( H^2 < C_s^2 K_z^2 \), that is, only when the speed of the low-frequency wave is smaller than the sonic speed, and if the wave is Alfvén wave, then the Alfvén velocity must be smaller than the sound velocity. Since \( \beta/2 = (C_s / V_A)^2 \), the self-modulation instability can only take place in the high-\( \beta \) region of the solar corona.

3.2 \( K_z = 0 \). This corresponds to a purely transverse perturbation. In this case, equation (10) becomes
\[
(\Omega - K_z V_s^2)^2 = \frac{1}{4} V_s^2 K_z^2 + \frac{G(\Omega, K)}{L (\Omega, K) H (\Omega, K)} T K_x^2 |\Psi_0|^2.
\]

This last equation can be discussed separately for \( T > 0 \) and \( T < 0 \).

For \( T > 0 \), the necessary condition for instability is the simultaneous satisfaction of the two inequalities,
\[
\Omega^2 - (V_s^2 + C_s^2) K_z^2 < 0,
\]
\[
d_1^2 K_z^2 + \left[ \Omega^2 - (V_s^2 + C_s^2) K_z^2 \right] \left( \frac{k \Omega}{\nu} d_1 \right)^2 > 0.
\]

(14a) can be realized in the excitation of slow magneto-acoustic waves; at the same time, because \( d_3 \geq d_2 \), (14b) is also satisfied. It is only when the amplitude of the wave \( |\Psi_0|^2 \) is very large that the right side of the equation can be negative to generate instability.

For \( T < 0 \), we only require
\[
\Omega^2 - (V_s^2 + C_s^2) K_z^2 > 0.
\]
that is, as long as the perturbing low-frequency wave velocity is greater than the magneto-acoustic velocity, instability will occur. And the requirement on the wave amplitude is also less in this case than in the case of $T > 0$.

3.3 $\Omega = \Omega_r + \Omega_i, \Omega_r = V_g K_z \gg \Omega_i$. This corresponds to an oblique wave perturbation. From (13) we see that the necessary condition for instability is

$$G(V_z K_z, K) H(V_z K_z, K) (V_z K_z + T K_z^2) > 0. \quad (16)$$

Again, we can discuss separately the cases $(V_z K_z + T K_z^2) > 0$ and $< 0$.

For $(V_z K_z + T K_z^2) > 0$, we have the inequalities,

$$(17a) \quad V_z K_z - (V_z^2 + c_s^2) K_x^2 - V_x K_x^2 < 0,$$

$$(17b) \quad d_z^2 K_x^2 H + L K_x^2 \left( d_z - \frac{k V_z}{\omega} \right)^2 > 0.$$ 

Because $V_z$ is much greater than $V_A$ or $C_s$, the condition $(17a)$ is satisfied only if $K_x^2 \ll K_z^2$, This is similar to the case of $T > 0$ in 3.2.

For $(V_z^2 K_z^2 + T K_z^2) < 0$, we also require $T < 0$, and it is similar to the case of $T < 0$ in 3.2. But from equations (3) and (4a) we see that, if $\omega_{pe} < \omega_{ce}$, then $T$ cannot be less than zero. Hence only the transverse self-modulation with $T > 0$ can take place.

In sum, when an electromagnetic wave propagates in the solar coronal plasma, if the perturbation is longitudinal, then self-modulation takes place only when the $\beta$ value is high (perturbing wave velocity less than sound velocity); if the perturbation is transverse, then we require the perturbing frequency to be very low and the perturbing amplitude, very high. We now apply these conditions in relation to the solar coronal parameters.

4. SOLAR CORONAL PLASMA PARAMETERS AND SELF-MODULATION INSTABILITY

From the discussion in the previous section, we see that the necessary condition for the occurrence of self-modulation depends on the parameters of the solar coronal plasma. Of the parameters, the density and temperature profiles are well determined[14], but the variation of the magnetic field cannot be accurately measured. Krüger[15] gave some estimates of the magnetic field from radio observations (see Fig. 1). The $\omega_{pe}/\omega_{ce}$ ratio can easily be found from Fig. 1 (where $\nu_p = 2\pi \omega_{pe}$). Generally speaking, the value of $\beta$ is always greater in the outer than in the inner corona. Taken as a whole, the value of $\beta$ throughout the corona is far less than 1 (magnetic frozen-in), but for different regions and times, the value of $\beta$ can vary: for example, it may rise at the time of flares in active regions. We now estimate $\beta$ for typical coronal parameters in meter burst sources. We take a temperature of $10^6$ K (or 100 ev), a density of $10^{10}$ cm$^{-3}$ (usual range $10^{10}-10^{11}$, according to Ref. [14]), and a magnetic field of 5 G (usual range 3–10 G, Ref. [15]). Then we have $\beta = 8\pi n kT/B^2 = 1.6 > 1$. Thus, in
localized regions at some particular instants of time, \( \beta \) can be greater than 1. This said, we now discuss the possibility of self-modulation instability in the corona.

1. The self-modulation instability discussed above cannot happen in the optical or X-ray ranges. This is because, for these waves, the frequency is far smaller than the coronal electron cyclotron frequency, \( u \gg 1 \), hence from (4a) and (4b), the group velocity of the wave is nearly equal to the sound velocity and the group velocity dispersion is nearly zero, hence, by (12), the instability cannot occur. Also, because \( u \gg 1, T > 0 \), transverse self-modulation instability cannot occur either.

2. By the same reasoning, for high-frequency waves generated in the inner corona, self-modulation instability will not occur easily. For in the lower corona, \( \omega_{pe} \ll \omega_{ce} \), or \( T > 0 \), so transverse instability is difficult, while because \( V_A^2 \gg c^2 \) so is the longitudinal instability. When the high-frequency reaches the outer corona, we have \( u \gg 1 \), self-modulation instability becomes even less likely, as stated above.

3. For low-frequency (decimeter and meter) waves generated in the outer corona, because of the weaker magnetic field and higher temperature, longitudinal instability will be possible, according to the above discussion. And if the wave amplitude \( |\Psi_0|^2 \) is very large

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Fig. 1 Some estimated results of coronal magnetic field from solar radio observation. In the figure, \( \nu_p \) is coronal plasma frequency, \( R' \) is distance from surface of photosphere (courtesy from [16])

Fig. 2 The ms envelope solitary fine structure in the event on the Aug. 23, 1990, observed from Yunnan Observatory meter radio spectrum (courtesy from [16])
and the wave velocity very small, then under the condition $T > 0$, transverse instability will also be possible.

4. The discussion in the last section shows that, whatever the type of instability, its occurrence requires a certain size of the wave amplitude. Hence, such instabilities can occur easily only during solar burst events. If we write $-\Delta \omega = Q|\psi_0|^2$ in the equation (12) for longitudinal instability, then the equation assumes the form of Schrödinger’s equation, and the solution is

$$\psi = \psi_m \text{sech} \left[ \frac{Q}{V_e} \right]^{1/2} s. \quad (18)$$

and this a typical envelope-type soliton solution.

5. Such soliton type fine structures in meter waves (290–297 MHz) were in fact observed by Xia Zhi-guo et al.\cite{16} in the burst event of 1990–08–23, the soliton period was 0.5 s. See Fig. 2. The shape of the structures can be seen to be precisely the shape described by (18). If these features did originate in the sun, then this type of fine structures can be interpreted as due to longitudinal self-modulation instability when a low-frequency electromagnetic wave propagates in the solar corona.

Of course, our model considers only simplified cases: the propagation of the wave may be inclined at any angle to the magnetic field and the plasma density in the corona may be varying continuously. These factors will affect the generation of self-modulation and should be further studied. Also, the dispersion equation used here, equation (3), holds only under the condition $\omega_{pe} < \omega_{ce}$: it should be modified and rediscussed in the contrary case.

References