The evolution of a beamed gamma-ray-burst afterglow: the non-relativistic case

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ABSTRACT

There has been increasing evidence that at least some gamma-ray bursts (GRBs) are emission beamed. The beamed GRB-afterglow evolution has been discussed by several authors in the ultrarelativistic case. It has been shown that the dynamics of the blast wave will be significantly modified by the sideways expansion, and there may be a sharp break in the afterglow light curves under certain circumstances. However, this is only true when the fireball is still relativistic. Here we present an analytical approach to the evolution of the beamed GRB blast wave expanding in the surrounding medium (density $n \propto r^{-s}$) in the non-relativistic case, our purpose is to explore whether the sideways expansion will strongly affect the blast-wave evolution as in the relativistic case. We find that the blast-wave evolution is strongly dependent on the speed of the sideways expansion. If it expands with the sound speed, then the jet angle $\theta$ increases with time as $\ln t$, which means that the sideways expansion has little effect on the afterglow light curves. It is clear that the light curve of $s = 2$ is not always steeper than that of $s = 0$, as in the relativistic case. We also show that if the expansion speed is a constant, then the jet angle $\theta \propto t$ and the radius $r \propto t^0$, in this case the sideways expansion has the most significant effect on the blast-wave evolution, the flux $F \propto t^{-(5\alpha - 1)}$, independent of $\alpha$, and we expect that there should be a smooth and gradual break in the light curve.

Key words: gamma-rays: bursts.

1 INTRODUCTION

The fireball model of gamma-ray bursts (GRBs) led to the prediction of the afterglow emission that might be expected when the energetic shock wave encountered the surrounding medium. The subsequent X-ray and optical observations of the afterglow appeared to confirm the predictions of the simplest afterglow model (Wijers, Rees & Mészáros 1997). This model involved synchrotron emission from electrons accelerated to a power-law energy spectrum in a relativistic blast wave. However, both geometry and environment can affect the evolution of GRB afterglows (Panaitescu, Mészáros & Rees 1998). There has been increasing evidence that at least some GRBs are emission beamed or that the surrounding medium is not uniform. A class of GRBs, the afterglows of which exhibited steeper than normal power-law decays ($f \propto t^{-\gamma}$), can be well explained by the jet-like geometry of the relativistic shock (Rhoads 1997, 1999; Sari, Piran & Halpern 1999; Wei & Lu 2000), or the inhomogeneous surrounding medium models (especially the wind model, Dai & Lu 1998; Chevalier & Li 1999; Li & Chevalier 1999). The jet model is also supported by the steepening of the optical and radio light curves seen in GRB 990510 (Harrison et al. 1999; Stanek et al. 1999).

The dynamical evolution of GRB fireballs and the emission features have been studied by many authors (e.g. Sari 1997; Mészáros, Rees & Wijers 1998; Sari, Piran & Narayan 1998; Wei & Lu 1998a,b, but most of them considered the fireball as being isotropic. The evolution of the beamed blast wave was first discussed by Rhoads (1997, 1999). He has shown that if the lateral expansion of the relativistic plasma causes, at some moment, the surface of the blast wave to begin to increase faster than it would do as a result of the cone-outflow alone, then the blast wave will begin to decelerate faster than it would do without the sideways expansion because more interstellar medium has been swept up by the blast wave. He claimed that this effect will produce a sharp break in the GRB-afterglow light curves. More detailed calculations, in both numerical and analytical studies (Wei & Lu 2000; Moderski, Sikora & Bulik 2000), have found that unless the

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opening angle is very small, or that lateral expansion is unimportant, a smooth and gradual transition is expected. However, these results are valid only when the blast wave is relativistic. As shown by Rhoads (1999) and Panaitescu & Mészáros (1999), the Lorentz factor $\Gamma_b$ at the radius $r_b$ where the sideways expansion becomes important is $\Gamma_b \sim \left(2/\sqrt{3}\right)\theta_0^{-1}$, where $\theta_0$ is the initial half-opening angle of the jet. To keep $\Gamma_b \gg 1$ requires that $\theta_0 < 0.1$, so that if the jet angle is not too small then the blast wave will become non-relativistic when sideways expansion is important, and the previous results are invalid.

Frail, Waxman & Kulkarni (2000) reported on the results of an extensive observation of the radio afterglow of GRB 970508, which lasted 450 d after the burst. They have shown that the spectral and temporal radio behaviour indicates that the fireball has undergone a transition to non-relativistic expansion at $t \sim 100$ d, and they find that the fireball may be initially a wide-angle jet of opening angle $\sim 30^\circ$. Therefore it is very interesting and important to study the behaviour of the beamed blast wave in the non-relativistic case. Recently Huang et al. (2000) have calculated the evolution of jetted GRB ejecta numerically.

Here we present an analytical approach to the evolution of the beamed GRB afterglow in the non-relativistic case, including both homogeneous and wind-shaped media (the density $n \propto r^{-2}$), the main purpose is to explore whether the sideways expansion will strongly modify the blast-wave evolution and the afterglow light curve behaviour as it does in the relativistic case. In the next section we consider the dynamical evolution of the blast wave, in Section 3 we calculate the jet emission and afterglow light curve analytically, and finally we give some discussions and conclusions.

2 DYNAMICAL EVOLUTION OF THE JET

Now we consider the evolution of an adiabatic blast wave expanding in the surrounding medium. Assuming that the medium density $n \propto r^{-4}$, $s = 0$ corresponds to the homogeneous medium and $s = 2$ corresponds to the wind-shaped medium. For completeness, we first outline the results of evolution in the relativistic case here.

2.1 Relativistic case

For energy conservation, the evolution equation of the blast wave is

$$\Gamma^2 N = \text{constant},$$

(1)

where $\Gamma$ is the bulk Lorentz factor and $N$ is the total number of baryons swept up by the blast wave, $N \propto r^{3-\gamma}(1 - \cos \theta) \propto r^{3-\gamma} \theta^2$ for $\theta \ll 1$ and $\theta_0 = \theta_0 + \theta$, where $\theta_0$ is the initial jet opening half angle, $\theta$ describes the lateral expansion, which can be simply written as $\theta \sim v_s t_{\text{c}}/ct$. $v_s$ is the expanding velocity of ejecta material in its comoving frame and $t_{\text{c}}$ is the time measured in the burst frame (comoving frame). For relativistic expanding medium it is appropriate to take $v_s$ to be the sound speed $v_s = c/\sqrt{3}$. Because the jet expands relativistically, we must consider the relation $T \propto r^{1/2}$, where $T$ is the time measured in the observer frame. According to Wei & Lu (2000), for $s = 0$, we obtain

$$\Gamma \propto \begin{cases} T^{-3/8} \left[1 + \left(\frac{T}{T_b}\right)^{3/8}\right]^{-1/4}, & \text{if } T < T_b \\ T^{-3/8} \left[1 + \left(\frac{T}{T_b}\right)^{1/2}\right]^{-1/4}, & \text{if } T > T_b, \end{cases}$$

(2)

where $T_b$ is the time at the moment when the sideways expansion is important. Similarly, for $s = 2$, we have

$$\Gamma \propto \begin{cases} T^{-1/4} \left[1 + \left(\frac{T}{T_b}\right)^{1/4}\right]^{-1/2}, & \text{if } T < T_b \\ T^{-1/4} \left[1 + \left(\frac{T}{T_b}\right)^{1/2}\right]^{-1/2}, & \text{if } T > T_b, \end{cases}$$

(3)

so we see that for $T \ll T_b$, $\Gamma \propto T^{-3/8}$ and $r \propto T^{1/4}$ for $s = 0$, and $\Gamma \propto T^{-1/4}$ and $r \propto T^{1/2}$ for $s = 2$. Whereas for $T \gg T_b$, both $s = 0$ and $s = 2$ give $\Gamma \propto T^{-1/2}$ and $r \propto T^0$.

2.2 The non-relativistic case

The evolution of the jetted blast wave in the non-relativistic case is $\beta^2 N = \text{constant}$.

(4)

where $\beta$ is the velocity of the blast wave in units of light speed $c$. Thus, for $\theta \ll 1$, we have $\beta \theta^2 = \text{constant}$. In this case the jet angle $\theta_j$ can be written as $\theta_j = \theta_0 + \int \beta \, dt / \{\beta \, dr\}$, where $\beta_s$ is the speed of ejecta material in units of light speed $c$, which cannot be simply determined. Here we consider several situations.

2.2.1 $\beta_s = \text{sound speed}$

Kirk & Duffy (1999) have derived the sound speed in the fluid to be

$$c_s^2 = \frac{P}{\rho} \left[\frac{\gamma - 1}{\gamma - 1} \rho + \frac{\gamma P}{\gamma - 1}\right],$$

(5)

where $P$ is the pressure, $\rho$ is the mass density and $\gamma$ is the adiabatic index. According to Huang et al. (2000), the sound speed can be written as

$$c_s^2 = \frac{\gamma (\gamma - 1)}{1 + \gamma (\gamma - 1)},$$

(6)

In the non-relativistic limit ($\gamma = 1$, $\dot{\gamma} = 5/3$), one gets $c_s = \sqrt{5}\beta/3$, then we obtain the jet angle $\theta_j = \theta_0 + \{2 \times \sqrt{5}/[3(5 - 3)] \ln(t/t_0)\}$, where $t_0 \approx t_{\text{NR}}$, $t_{\text{NR}}$ is the time when the blast wave turns from relativistic to non-relativistic. Therefore we see that because the jet angle $\theta_j$ increases with time as the log relation, it will not strongly affect the evolution of the jet, and the variation of the jet velocity is about $\beta \propto t^{-3/5}$ for $s = 0$ and $\beta \propto t^{-1/3}$ for $s = 2$.

2.2.2 $\beta_s = \text{constant}$

If the jet spreads at a constant speed then, for $t \gg t_{\text{NR}}$, the angle $\theta_j = \theta_0 + [2/(3 - s)](\beta_s/\beta)$, and the evolution of $\beta$ satisfies $\beta^{3-s} (\theta_0 + [\beta_S/\beta]^2 \propto r^{-3-s}$, so when $\theta \ll [2/(3 - s)]\beta_03^{-1}$, we get $\beta \propto r^{-1}$ and $r \sim \text{constant}$, independent of $s$. We see that in this case it is similar to the situation in which sideways expansion is important in the relativistic case.

2.2.3 In general

We assume that the ratio of the jet spread velocity and the blast wave velocity varies with time, $\beta_s/\beta = a(t)$. We then obtain, for $t \gg t_{\text{NR}}$, the jet angle $\theta_j = \theta_0 + [2a(t)/(5 - s)]t^s$, and

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for \( t \gg [(5 - s)\beta_0^2/2a]^1/3 \), the evolution of \( \beta \) satisfies \( \beta \propto t^{-(3-s-2\alpha)/(5-s)} \), so \( \beta \propto t^{-(3+2\alpha)/3} \) for \( s = 0 \) and \( \beta \propto t^{-(1+2\alpha)/3} \) for \( s = 2 \). We see that \( q = 1 \) corresponds to the case described in Section 2.2.2. Therefore, it seems that the evolution of the blast wave strongly depends on the parameter \( q \).

3 THE EMISSION FROM THE JET

Now we calculate the emission flux from the non-relativistic jet. Here we adopt the formulation and notations of Mao & Yi (1994). In our model the ejecta is flowing outwards in a cone with an opening half angle \( \theta_0 \). For simplicity, we assume that the radiation is isotropic in the comoving frame of the ejecta and has no dependence on the angular positions within the cone. The radiation cone is uniquely defined by the angular spherical coordinates \((\theta, \phi)\), here \( \theta \) is the angle between the line of sight (along the \( z \)-axis) and the symmetry axis, and \( \phi \) is the azimuthal angle. Because of cylindrical symmetry, we can assume that the symmetry axis of the cone is in the \( y-z \) plane. In order to see more clearly, let us establish an auxiliary coordinate system \((x', y', z')\) with the \( z' \)-axis along the symmetry axis of the cone and the \( x' \)-axis parallel with the \( x \)-axis. Then the position within the cone is specified by its angular spherical coordinates \( \theta' \) and \( \phi' \) (\( 0 \leq \theta' \leq \theta_0, 0 \leq \phi' \leq 2\pi \)). It can be shown that the angle \( \Theta \) between a direction \((\theta', \phi')\) within the cone and the line of sight satisfies \( \cos \Theta = \cos \theta \cos \theta' - \sin \theta \sin \theta' \cos \phi' \). Then the observed flux is

\[
F(\nu, \theta) = \int_0^{2\pi} \int_0^{\theta_0} \sin \theta' d\theta' d\phi' \cos \Theta D^3 \nu D^{-1} \sin^2 \theta, \tag{7}
\]

where \( D = \left[ I(1 - \beta \cos \Theta) \right]^{-1} \) is the Doppler factor, \( \beta = (1 - \gamma^{-2})^{1/2} \), \( \nu = D^2 \nu' \), \( I(\nu') \) is the specific intensity of synchrotron radiation at \( \nu' \) and \( d \) is the distance of the burst source. Here the quantities marked with a prime are measured in the comoving frame. For simplicity we have ignored the relative time delay of radiation from different parts of the cone.

For the non-relativistic blast wave, it is well-known that the accelerated electrons have a power-law energy distribution \( n(\gamma) \propto \gamma^{-p} \) in the range \( \gamma_1 \leq \gamma \leq \gamma_2 \), then the typical electron energy is given by \( \gamma_2 = (1/2)e_e(p-2)/(p-1)(m_e/m_e)^{\beta_0^2} \), where \( e_e \) is the energy fraction occupied by the electrons, \( m_e \) and \( m_e \) are the mass of a proton and an electron, respectively. Assuming that \( e_e \) is a constant, we have \( \gamma_1 \propto \beta_0^2 \). For the non-relativistic case, \( \Gamma - 1, \beta = D \beta_0 \propto \beta_0 \), \( \beta_0 \propto H^{-1/2} \approx \beta_0^{1/2} \) and the peak frequency of synchrotron radiation \( \nu_m = D^2 \nu_m \propto D^2 \gamma_0^2 \beta_0^2 n_0^{1/2} / (1 - \beta \cos \Theta) \), the energy density \( u \propto n_0 \beta_0^2 \), the magnetic field strength \( B \propto u^{1/2} / \beta_0 n_0^{1/2} \) and the peak frequency of synchrotron radiation \( \nu_m = D^2 \nu_m \propto D^{-2} \beta_0 \beta_0^{2/1/2} n_0^{1/2} / (1 - \beta \cos \Theta) \), Assumption that the emission spectrum \( I(\nu') \propto \nu'^{-\alpha}, I(\nu') = I(\nu') (\nu')^{-\alpha} \propto \beta^{2+\alpha} n_0^{\alpha}(3+\alpha)I(\nu') / (1 - \beta \cos \Theta)^{\alpha} \), therefore we have the emission flux

\[
F(\nu, \theta) \propto \nu^{-\alpha} \beta^{2+\alpha} n_0^{\alpha} I(\nu') g(\theta, \beta, \alpha), \tag{8}
\]

where

\[
g(\theta, \beta, \alpha) = \int_0^{2\pi} \int_0^{\theta_0} \sin \theta' d\theta' d\phi' \cos \Theta (1 - \beta \cos \Theta)^{-\alpha} \cos \phi' \tag{9}
\]

In general, the value of \( g \) can only be calculated numerically. However, here we consider the case \( \theta_0 \ll 1 \) and \( \theta \ll 1 \), then \( \cos \Theta = \cos \theta \cos \theta' \). In this case we can calculate the value of \( g \) analytically under certain conditions. After complicated calculation we find \( g = \cos \theta \sin^2 \theta \), so finally we obtain

\[
F(\nu, \theta) \propto \nu^{-\alpha} \beta^{2+\alpha} n_0^{(3+\alpha)/2} I(\nu') \cos \theta \sin^2 \theta, \tag{10}
\]
for the case in Section 2.2.2, i.e. the sideways expansion is very important, we find that the light curve is independent of \( s \) as in this case \( r \sim \text{constant} \).

Recently Frail et al. (2000) have reported on the results of 450-d observations of the radio afterglow of GRB 970508, and indicated that the fireball has undergone a transition to non-relativistic expansion at \( t \sim 100 \text{d} \). Therefore, it is very interesting and important to explore the dynamical evolution of the non-relativistic jetted material in various surrounding media, because we expect that over the duration of the radio emission, the emitting material will typically become non-relativistic.

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**REFERENCES**


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