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Investigation of intermittent magnetic flux in the auroral zones with kilometer radiation (AKR)

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On the basis of the nonlinear equations for self-generated magnetic fields, it is numerically shown that the magnetic fields self-generated are instable and may collapse, resulting in spatially highly intermittent flux fragment. Numerical results show that the enhanced magnetic flux has a strength about up to $10^{-2}$ Gauss in range about around 250–350 km in auroral zones with kilometric radiation (AKR), which correspond to estimated values in both the strength and characteristic scale by Mckean et al. [J. Geophys. Res. [Oceans] 96, 21055 (1991)]. © 2001 American Institute of Physics. [DOI: 10.1063/1.1337067]

I. INTRODUCTION

Nearly all cosmic bodies have magnetic fields. The fields observed in galaxies, stars or planets, which are important for the motion, structure and evolution of the bodies, are large-scale macromagnetic fields. The fields are believed to result from the processes of magnetohydrodynamics (MHD), that are responsible for large-scale global properties. Stellar matter has high conductivity, which leads to a very long decay time for the fields, but then because of active magnetic features in celestial bodies, for instance, the reversals of polarity for solar and terrestrial magnetic fields and variations in magnetic fields for stars, the origin of the fossil cannot explain the observed nonstationary fields of the earth, the sun, convective stars, and some other objects. So far as neutron stars are concerned, this case is also true. Analysis of the rotation period and its changes for many pulsars has shown that majority of neutron stars do not become pulsars until long after their formation.1 It has been shown that there is a need for a search for the field generation processes for neutron stars.2 The dynamo theory is the most elaborated theory of large-scale field generation, which claims that large-scale fields grow from arbitrarily weak but finite seed magnetic fields as they couple with a turbulent fluid. This kinematic dynamo has been successful at least in the discussion of such events as solar and stellar activity cycles, but this is hardly surprising, since the dynamo theory contains so many free and adjustable parameters. It cannot exclude that some successes of the dynamo in application to concrete objects are based on the uncertainty of the parameters.3 A major deficiency of the current dynamo theory is the absence of nonlinear interaction, so it should be pointed that the study of nonlinear dynamo is clearly necessary.4

On the other hand, besides the large-scale magnetic field, cosmic bodies have small scale nonuniform fields, which cannot be resolved by present day instruments. The sun is a typical sample. All magnetic structures on the sun, excluding sunspots, are smaller than can be explored directly;5 it is shown that spatially highly intermittent flux fragments can occur all over the sun. There is now a considerable body of evidence suggesting that all scales of structure in the solar corona, as well as other astrophysical interesting objects, are coupled to small-scale processes associated with intermittent magnetic fields. Present models for the site of primary energy release, particle acceleration and emission all involve mechanisms that require gradients in the magnetic field on scales in the range (0.01–10) km.

Auroral kilometric radiation (AKR) is an intense electromagnetic emission originated in the auroral zones, which does not reach ground-based observatories and can be only observed by high-altitude Earth-orbiting satellites carrying electric antennas. AKR is characterized by complex temporal and frequency fine structure. The frequency fine structure consists of many narrow-band emissions with rapidly varying center frequencies; the center frequencies can either rise or fall with time, drifting at rates varying from a few hundred hertz per second to a few tens of kilohertz per second. Several theories have been proposed for the interpretation of the frequency fine structure of AKR. Burnett6 and later Melrose7 suggested that the frequency drift is caused by the movement of the source region, in which there are electrostatic perturbations. Though many possibilities exist for the nature of these disturbances, none have been confirmed. Models that rely on the density inhomogeneities, first proposed by Calvert8 and later expanded by Le Queau,9 Zarka,10 Le Queau and Louarn11 suggest that frequency drift is a result of density fluctuations. But no corresponding density fluctuations have been found. Mckean and Winglee12 investigated the source of AKR frequency fine structure by using an electromagnetic, particle-in-cell code to simulate auroral plasma. In the study the frequency drift is determined predominantly by the nonuniformity of the magnetic field rather than

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changes in the emitting electron distribution function with time. In the simulation model, this enhanced magnetic field in the emission region has characteristic scale \( l_c = 88c/\omega_{pe} \) (\( c \) is the velocity of light and \( \omega_{pe} \) is the plasma frequency). The nonuniform magnetic field is chosen so that \( \omega_{pe} / \Omega_e \) (\( \Omega_e \) is the electron gyrofrequency) is about 0.2. Outside this region the magnetic field decreases rapidly to the background value, which corresponds to magnetic field value in solar wind at that location, or to the value of Earth’s magnetic field, which is about \( B_e = (10^{-3} - 10^{-2}) \) Gauss as a magnetic dipole at the same location.

As the matter above stands, one of the more interesting and unresolved problems is posed: How is such a highly intermittent magnetic field on a small scale generated in self-consistent manner and developed? Here obviously, magnetohydrodynamic (MHD) processes, say convection and rotation, are not relevant. The fields are believed to be related to localized instabilities in plasma, so to investigate, generally speaking, the nonlinear self-generated magnetic fields in detail is necessary.

It is well known that a plasma is a system with a large number of degrees of freedom; in such a highly unstable plasma, the tendency for energy equilibration over the different possible degrees of freedom can produce turbulent waves, that is to say, plasmons are excited at a rather high level. In a plasma there is a transverse mode, with frequencies \( \omega^p \) which are nearly \( \omega_{pe} \).

\[
\omega^p = \omega_{pe} + \frac{k^2 c^2}{2 \omega_{pe}} (\omega_{pe} \gg k c),
\]

or

\[
\omega^p = \omega_{pe} + \frac{3k^2 v_T^2}{2 \omega_{pe}} \left( k = \frac{\sqrt{3} v_T c}{k} \right),
\]

their group velocities, like those of Langmuir waves, are very small (\( v^p \ll c \)). It is extremely difficult for the oscillations to escape the plasma because the index of refraction for these waves is very nearly zero. Hence the excitation of transverse waves does not mean that there will be radiation from the plasma at frequencies near \( \omega_{pe} \). This is one of the reasons why Langmuir and the transverse mode are often grouped together and called plasma oscillations. Thus, it is convenient to call the transverse mode of Eq. (1) transverse plasmons. Due to very small group velocities, the interactions between the transverse plasmons and Langmuir waves are strong, the interactions are connected with scattering by electrons and ions \( l + e = p + e', l + i = p + i' \) and the decay processes \( l + i' = p \). The numerical calculations show that there is a continuous transfer from Langmuir waves to the transverse plasmons and back and their energy densities are approximately the same, averaged over time, \( W^l \approx W^p \). In view of the physics, it is natural that the interactions will lead to tendency of an equilibration of energy over both of Langmuir and transverse plasmons with the same frequencies near \( \omega_{pe} \) and similar dispersion laws.

On the other hand, for a plasma in thermal equilibrium, there is also a finite level of plasma waves, which represents degrees of freedom excited in thermal equilibrium. Langmuir plasmons are excited by the charged particles of the plasma as they move due to their thermal energy by the Cerenkov processes \( e \rightarrow l + e' \), and they are then reabsorbed by the plasma due to Landau damping. A balance between spontaneous emission and induced absorption leads to a thermal level of Langmuir plasmons. As a result, we may expect that for a plasma on excited levels of the modes, \( \omega_{pe} / \Omega_e \)

\[
\tilde{W}_p = \frac{W^p}{n_e T_e} > \frac{1}{N_D},
\]

where \( W^p \) is the energy density of the transverse plasmons, \( n_e \) the density of electrons, \( T_e \) the temperature (in units of energy), and \( N_D = n_e(v_{Te}/\omega_{pe})^3 \) is the Debye number.

References 14 and 16 were devoted to the study of self-generated magnetic fields by transverse plasmons with dispersion relation (1). In a plasma with a Maxwellian distribution, coupling of the transverse plasmons may also produce low-frequency fields and nonlinear currents, leading to the creation of magnetic fields. It is shown analytically that self-generated fields are modulationally unstable, resulting in strong localized magnetic structures flux;\(^1^7\),\(^1^8\) but the linear analyses cannot give flux strength values in these studies. In this paper, we try to examine numerically the collapse behavior of the self-generated magnetic fields with relevant initial values, resulting in the localized magnetic structures for AKR and coronal active regions, in both the kilometric radiation and the coronal active regions where there are very highly ionized and collisionless plasmas with rather weak background magnetic fields relative to the resulting fields. Therefore, it is appropriate to use Vlasov and Maxwell equations to depict the wave–particle interactions and wave–wave interactions in the both zones.

II. NONLINEAR EQUATIONS FOR THE FIELDS

We start from the Vlasov equation,

\[
\frac{\partial f_{\alpha}}{\partial t} + v \frac{\partial f_{\alpha}}{\partial x} + \mathbf{F} \frac{\partial f_{\alpha}}{\partial p} = 0 \quad (\alpha = e, i),
\]

\[
\mathbf{F} = e \left[ \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right],
\]

where \( f_{\alpha} \) is the distribution function of charge particle,

\[
\int f_{\alpha} \frac{dp}{2 \pi^3} = n_{\alpha},
\]

where \( n_{\alpha} \) is the density of particles and \( \mathbf{F} \) is the electromagnetic force. One can divide \( f_{\alpha} \) and \( \mathbf{F} \) into two parts, the unperturbed and perturbed, \( f_{\alpha} = f_{\alpha}^0 + f_{\alpha}^T \), \( \mathbf{F} = \mathbf{F}^0 + \mathbf{F}^T \).

As \( f_{\alpha} \) and \( \mathbf{F} \) are closely coupled through Maxwell’s equation and the current density equation

\[
\mathbf{j} = \sum_{\alpha} \int e_{\alpha} v f_{\alpha} \frac{dp}{2 \pi^3},
\]

\[
\tilde{W}_p = \frac{W^p}{n_e T_e} \gg \frac{1}{N_D},
\]
and the field $\mathbf{E}$ is assumed to be weak, so that the energy density excited is much smaller than thermal one of the plasma, i.e.,
\begin{equation}
\tilde{W} = \frac{E^2}{8 \pi n_e T_e} \ll 1,
\end{equation}
the perturbed distribution $f^T_a$ can be expanded in powers of the perturbed field $E^T$,\begin{equation}
 f^T_a = \sum_a f^{T(\alpha)}_a,
\end{equation}
where the index $a$ indicates that $f^{T(\alpha)}_a$ is proportional to the $\alpha$th power of $E^T$. Putting $F^0 = 0$ and expanding $A = (F, f)$ in a Fourier series,
\begin{equation}
 A(r, t) = \int A_k e^{-i\omega t + ikr} dk \quad (A_k = A_{k, \omega} dk = dk \cdot d\omega),
\end{equation}
and taking Eqs. (8), (10), and Maxwell equations into consideration, we get the following field equation for the transverse mode from Eq. (4):
\begin{equation}
 (k^2 c^2 - \omega^2 \varepsilon'_k) E^T_k = 4 \pi i \omega \varepsilon'_k \cdot (j^{(2)}_k + j^{(3)}_k),
\end{equation}
where $\varepsilon'_k$ is the transverse dielectric constant,
\begin{equation}
 \varepsilon'_k = 1 + \frac{2 \pi \varepsilon_0}{\omega k^2} \int \frac{(k \times v)^2}{\omega - k \cdot v + i \varepsilon} \frac{\partial f^R_a}{\partial \mathbf{p}} \frac{d\mathbf{p}}{(2 \pi)^3},
\end{equation}
where $\varepsilon_0$ is the particle energy.

We can obtain perturbed distribution functions from Eqs. (4) and (10) (to the second and the third order),
\begin{equation}
 f^{T(2)}_k = e^2 \int \frac{\hat{e}'_k \cdot \partial \mathbf{p}}{i(\omega - k \cdot v + i \varepsilon)} E^T_k E^T_{k_1} \delta(k - k_1 - k_2) dk_1 dk_2,
\end{equation}
\begin{equation}
 f^{T(3)}_k = e^3 \int \frac{i \hat{e}'_k \cdot (\partial \mathbf{p})}{\omega - k \cdot v + i \varepsilon} \frac{(\hat{e}'_k \cdot (\partial \mathbf{p}))}{(\omega - \omega_1) - (k - k_1) \cdot v + i \varepsilon} \frac{\hat{e}'_k \cdot (\partial \mathbf{p})}{(\omega - \omega_2) - (k - k_2) \cdot v + i \varepsilon} \frac{\hat{e}'_k \cdot (\partial \mathbf{p})}{\omega_3 - k_3 \cdot v + i \varepsilon} \frac{E^T_{k_1} E^T_{k_2} E^T_{k_3}}{\omega_3} \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3,
\end{equation}
where $\hat{e}_k$ is the polarization vector for the transverse mode
\begin{equation}
 \hat{e}_k = \frac{1 - k \cdot v/\omega}{k_0} e_0 + (e_0 \cdot v) k/\omega,
\end{equation}
\begin{equation}
 \delta(k - k_1 - k_2) = \delta(k - k_1 - k_2) \delta(\omega - \omega_1 - \omega_2).
\end{equation}
For the low-frequency fields $E^T_k = E^T_i$, substituting Eqs. (14) and (8) into Eq. (12), only retaining the dominant contributions of electrons, we obtain
\begin{equation}
 (k^2 c^2 - \omega^2 \varepsilon'_k) E^T_k
 = 4 \pi i \omega \int \tilde{S}^{(t)}_{k, k_1, k_2} E^{T(+) T^{(-)} T^{(-)} T^{(+) T^{(-)}}}_{k_1} \delta(k - k_1 - k_2) dk_1 dk_2,
\end{equation}
For the high-frequency field $E^T = E^T_j$, taking both the second and the third order of nonlinear current into account, we similarly obtain
\begin{equation}
 (k^2 c^2 - \omega^2 \varepsilon'_k) E^T_k
 = 4 \pi i \omega \int \tilde{S}^{(t)}_{k, k_1, k_2} E^{T(+) T^{(-)} T^{(-)} T^{(+) T^{(-)}}}_{k_1} \delta(k - k_1 - k_2) dk_1 dk_2.
\end{equation}
where $\tilde{S}^{(t)}_{k, k_1, k_2}, \tilde{S}^{(t)}_{k, k_1, k_2},$ and $\tilde{S}^{(t)}_{k, k_1, k_2}$ are matrix elements of interaction between the high-frequency field and low-frequency fields, and the upper indices ‘+’ and ‘—’ denote the positive and negative frequencies parts for the high-frequency perturbation, respectively, and $E^T$ in Eq. (20) is different from $E^T$ in Eq. (19) by a phase factor, and
\begin{equation}
 \tilde{S}^{(t)}_{k, k_1, k_2} = -e^3 \int \frac{(e_0 \cdot v)}{\omega - k \cdot v + i \varepsilon} \left( \hat{e}'_k \cdot \frac{\partial}{\partial \mathbf{p}} \right) E^T_k \delta(k - k_1 - k_2) \frac{d\mathbf{p}}{(2 \pi)^3},
\end{equation}
\begin{equation}
 \tilde{G}^{(t)}_{k, k_1, k_2, k_3} = ie^4 \int \frac{(e_0 \cdot v)}{\omega - k \cdot v + i \varepsilon} \left( \hat{e}'_k \cdot \frac{\partial}{\partial \mathbf{p}} \right) \frac{1}{(\omega - \omega_1) + (k - k_1) \cdot v + i \varepsilon} \frac{1}{(\omega - \omega_2) + (k - k_2) \cdot v + i \varepsilon} \frac{1}{(\omega - \omega_3) + (k - k_3) \cdot v + i \varepsilon} \frac{E^T_{k_1} E^T_{k_2} E^T_{k_3}}{\omega_3} \delta(k - k_1 - k_2 - k_3) \frac{d\mathbf{p}}{(2 \pi)^3},
\end{equation}
If the characteristic scale of the low-frequency field larger than the Debye length $d_\omega (\gg d_\omega)$, in the coordinate representation, we obtain
\begin{equation}
 \left( \frac{\partial^2}{\partial t^2} - v_\omega^2 \nabla^2 \right) n'(r, t) = \nabla^2 \frac{|\mathbf{E}(r, t)|^2}{4 \pi n_i},
\end{equation}
\begin{equation}
 \frac{2i}{\omega_p e} \frac{\partial E(r, t)}{\partial t} - \frac{c^2}{\omega_p^2} \nabla \times \nabla \times \mathbf{E}(r, t)
 = \frac{n'(r, t)}{n_0} \mathbf{E}(r, t) + \frac{ie}{m_e c \omega_p} \mathbf{E}(r, t) \times \mathbf{B}'(r, t),
\end{equation}
\[
\left(-\frac{\partial^2}{\partial t^2} + v_i^2 \nabla \nabla\right) B'(r,t)
\]
\[
= \frac{\gamma e c}{m_e \omega_{pe}^2} \nabla \nabla \times \frac{\partial}{\partial t} [E(r,t) \times E^*(r,t)],
\]
where \(n'(r,t)\) is perturbed density. Through the substitutions
\[
\xi = \frac{2}{3} \sqrt{\frac{1}{\mu}} \Gamma, \quad \tau = \frac{2}{3} \mu \omega_{pe} t, \quad \mu = \frac{m_e}{m_i},
\]
\[
\alpha = \frac{c^2}{3 \nu_T^2}, \quad n = \frac{3}{4 \mu \nu_0},
\]
\[
E(\xi, \tau) = \frac{\sqrt{3} E(r,t)}{4(\pi \mu_0 \nu_0 \nu_T)^{3/2}}, \quad B(\xi, \tau) = \frac{3 e}{4 \mu m_e c \omega_{pe}} B'(r,t),
\]
we can now write Eqs. (23)–(25) in the form
\[
\left(\frac{\partial^2}{\partial \tau^2} - \nabla^2 \right) n(\xi, \tau) = \nabla^2 \left| E(\xi, \tau) \right|^2, \quad \text{(28)}
\]
\[
i \frac{\partial}{\partial \tau} E(\xi, \tau) - \alpha \nabla \nabla \nabla E(\xi, \tau)
\]
\[
= n(\xi, \tau) E(\xi, \tau) + i E(\xi, \tau) \times B(\xi, \tau), \quad \text{(29)}
\]
\[
\left(-\frac{\partial^2}{\partial \tau^2} + \nabla \nabla \right) B(\xi, \tau)
\]
\[
= \frac{i}{3} \nabla \nabla \nabla \times \frac{\partial}{\partial \tau} \left[ (E(\xi, \tau) \times E^*(\xi, \tau)) \right].
\]
In the subsonic approximation, we can neglect the first term on the left-hand of Eqs. (28) and (30), in which case Eq. (28) is reduced to
\[
n = -|E|^2. \quad \text{(31)}
\]
Substituting Eq. (31) into Eq. (29) yields
\[
i \frac{\partial}{\partial \tau} E - \alpha \nabla \nabla \nabla E + |E|^2 E - i E \times B = 0, \quad \text{(32)}
\]
and Eq. (30) is reduced to
\[
B = \frac{2}{3} \frac{\partial}{\partial \tau} (E E^*). \quad \text{(33)}
\]

### III. NUMERICAL RESULTS

In our paper, for Eq. (32), that is, a sort of soliton-like equation with very steep gradients, following Ref. 21, we numerically calculate Eqs. (32) and (36) using the fast Fourier translation (FFT) method in Fourier space, and then obtain the numerical results in the time–space coordinate through the inverse translation, shown in Figs. 1–3. In Figs. 1–3, there are 80 points in the \(x\)-direction and 80 points in the \(y\)-direction in our numerical calculations problem with periodic boundary-conditions with respect to \(y\) is considered,

\[
E(\xi, \tau = 0) = E_0 \sin \left(\frac{2 \pi y}{y_0}\right) \text{sech} \left(\frac{x}{L_0}\right) (e_x + e_y)
\]
\[
- E_0 \frac{y_0}{\pi L_0^2} \cos \left(\frac{2 \pi y}{y_0}\right) \text{th} \left(\frac{x}{L_0}\right) \text{sech} \left(\frac{x}{L_0}\right) e_y,
\]
where \(y_0\) is a period and \(L_0\) is the width of the electromagnetic envelope. Figure 1 describes the distribution of the initial electric field. The evolutions of the solution of Eqs. (32)–(33) with the initial conditions (34) for AKR and coronal active regions are given in Figs. 2 and 3. The level contours of \(|B|^2 = B_x^2 + B_y^2 + B_z^2\) at successive times are also shown in Figs. 2 and 3.

Quantities in Figs. 1–3 are dimensionless. For auroral zones with kilometric radiation, their relations to dimensional ones are (taking \(n_e = 5 \text{ cm}^{-3}\), \(T_e = 6 \times 10^5 \text{ K}, \alpha = 3300,\)

\[
B = 5.17 \times 10^{-6} (B)_{\text{Fg}} (G),
\]
\[
|E|^2/(4 \pi n_e T_e) = 7.23 \times 10^{-4} (E)_{\text{Fg}}^2,
\]
\[
X = 1.49 \times 10^5 (x)_{\text{Fg}} (\text{cm}), \quad t = 2.82 \times 10^{-2} \tau (\text{s}).
\]
and the RMA turbulent electric field is $E = 0.12 \text{ V/m}$; the period and width are chosen as $y_0 = 3600$ and $L_0 = 600$, Fig. 2 gives the resulting field strength and the characteristic size as $B_{\text{max}}^2 = 5.81 \times 10^6$ and $x = 2.16 \times 10^2$, respectively, using Eq. (35) to yield

$$B = 1.2 \times 10^{-2} \text{ Gauss, } \ell_c = 322 \text{ km.}$$

FIG. 2. The collapse development of self-generated magnetic field by transverse pump field with a subsonic approximation when $|E_{\text{max}}|^2 = 4.5$.

We take $|E_{\text{max}}|^2 = 6$ initially, i.e., $E = 0.16 \text{ V/m}$, and take $y_0 = 2500$ and $L = 900$, the resulting field strength and characteristic size are $B_{\text{max}}^2 = 7.85 \times 10^6$ and $x = 1.62 \times 10^2$ (see Fig. 3), i.e.,

$$B = 1.5 \times 10^{-2} \text{ Gauss, } \ell_c = 241 \text{ km.}$$

FIG. 3. The collapse development of self-generated magnetic field by transverse pump field with a subsonic approximation when $|E_{\text{max}}|^2 = 6$.

It is worth noting that Mckean et al.12 proposed a model for AKR. In their model, the formation of the frequency fine structure is dependent on the existence of an intermittent...
magnetic field, they inferred that the fields would have a characteristic scale of 250–350 km (corresponding to \( l_e = \frac{88c}{\omega_{pe}} \)) and the strength of \( 10^{-2} \, \text{Gauss} \) (corresponding to \( \omega_{pe}/\Omega_e = 0.2 \)) for AKR, which are similar to our results.

**IV. CONCLUSION**

From the above study, we arrive at the following conclusions. Low-frequency nonlinear plasma currents could be excited by the high-frequency oscillation via the wave–wave and the wave–particle interactions, leading to excitation of a very low-frequency magnetic field. It is shown that the dynamic behavior and configuration for the self-generated field is determined by Eqs. (28)–(30). The numerical solution for Eqs. (32)–(33) (subsonic approximation) shows that the magnetic field self-generated by a transverse plasma with \( \omega_0 = \omega_{pe} \) would collapse (see Figs. 2 and 3). In other words, due to the self-condensing, a stronger magnetic field could be produced in a small region.

In a paper by Rubenchik and Zakharov,\(^{19}\) they have suggested that strong Langmuir turbulence can be described by Zakharov equations. For supersonic collapse they find self-similar solutions. Their results of numerical calculations confirm the hypothesis about the self-similar nature of the collapse. In fact, Zakharov equations show the explosive growing behavior of the strength of electric field. Similar results had been obtained by Rudakov and Tsytovich.\(^{20}\) Similarly, the nonlinear Eqs. (32) and (33), which are similar to Zakharov equations and have collapse behavior, just describe the explosion of strength of self-generated magnetic fields due to the modulation instabilities.

We have examined the problem of the linear instabilities for Eqs. (32) and (33). It is shown analytically that the self-generated magnetic fields are modulationally unstable;\(^{15,17}\) such instabilities would localize the magnetic fields; this localized magnetic flux may well produce small-scale intermittent magnetic fields.

In addition, we have analyzed the nonlinear Eqs. (32) and (33). We can find the Lagrangian density corresponding to Eqs. (32) and (33), and the conserved quantities (the plasmon number, the momentum, and the energy) for these equations; using the conserved quantities, we have analytically proved that the self-generated magnetic fields have the self-collapse behavior, i.e., the fields will collapse to such a value in a finite time, and that the pump field becomes too strong for Eqs. (32) and (33) to hold true.\(^{16}\)

Now the numerical results show that the self-generated magnetic fields may collapse, leading to the growing strength value of the fields. The results coincide well with the previous theoretical analyses, even then the size of the maximum peak magnetic fields fits in with the characteristic size obtained by the linear analysis for the modunational instabilities,\(^{15,17}\) so it is obvious that our numerical calculations are correct and reasonable.

The results of numerical analyses have shown that when \( |E_{\text{max}}|_{t=0} \) increases while \( L_0 \) remains unchanged, the collapse for self-magnetic fields becomes fast; when \( L_0 \) increases while \( |E_{\text{max}}|_{t=0} \) remains unchanged, the collapse for the self-magnetic fields becomes slow.

When the time scales \( \tau_1 > 0.60 \) (see Fig. 2), \( \tau_2 > 0.45 \) (see Fig. 3), respectively, the field collapse rapidly and leads to a very strong field, then \( W = E^2/8\pi n_e T_e = 2m_0 |E_{\text{max}}|_{t=0}^2 / 3m_1, > 1 \), in this case the expansion Eq. (10) is no longer valid.

On the other hand, we have solved numerically Eqs. (32) and (33) with the following initial condition:

\[
E(\xi, t=0) = E_0 \exp \left( -\frac{x^2}{2L_0} \right) \cos \left( \frac{2\pi y}{y_0} \right) e_x - \frac{xy_0}{\pi L_0} \sin \left( \frac{2\pi y}{y_0} \right) e_y + \cos \left( \frac{2\pi y}{y_0} \right) e_z. 
\]  

(37)

The results show that when we select the different distribution of initial field, the final results in both the strength and scale of collapse for self-generated magnetic fields in AKR and coronal active regions are the similar, but the speed and pattern of collapse are different.

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