A Study of the Mechanism of Acceleration of $^3$He and Heavy Ions by Alfven Turbulence in Impulsive Flares† *

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Abstract Prompted by the observational result that in $^3$He-rich events the energetic $^3$He and heavy ions exhibit similar power-law distributions, we study by means of numerical solution of Fokker-Planck equation, the characteristics of the evolution of the distribution of ions after they are accelerated by Alfven turbulence. The results of computation show that the plasma density in the source region of acceleration and the energy density of Alfven turbulence play the chief role in the energy spectrum of the particles. For a plasma density $n = (0.1 - 1) \times 10^{10}$ cm$^{-3}$, a magnetic field $B = 50 - 100$ G and a turbulence energy density $0.4-2$ ergs cm$^{-3}$, the turbulent Alfven waves can accelerate $^3$He and heavy ions to the order of magnitude of $10$ MeV/nucleon then in about 1 second. The index of the energy spectrum is 2.0-3.5. The result of the theoretical calculation basically agrees with the observation.

Key words: He-rich event—turbulent acceleration—energy spectrum of particles

1. INTRODUCTION

As shown in many observed solar energetic events $^{1-4}$, abnormal increases of the abundance of $^3$He and heavy ions are always associated with impulsive type flares. Compared to flares of slow variation type, the $^{3}$He/$^{4}$He ratio can be increased up to $10^3\times 10^4$ times among energetic...
slow variation type, the \(^3\)He/\(^4\)He ratio can be increased up to \(10^3 - 10^4\) times among energetic particles with energies of the order of MeV/nucleon, and the abundance of heavy ions (e.g., \(\text{Fe}/\text{O}\)) can be increased by a factor of 2–10. Although there is no intrinsic relation between the abundances of \(^3\)He and the heavy ions, they both exhibit a power-law distribution with very similar indices \(^5\). This indicates that \(^3\)He and the heavy ions are accelerated by one and the same mechanism.

Nowadays, probing the mechanism of the abundance of energetic \(^3\)He and heavy ions has become a hot topic in the theoretical research of particle acceleration. The existing theoretical models can be divided into two classes: one-step acceleration and two-step acceleration. The mechanism of one-step acceleration was first proposed by Temerin et al.\(^6\), and it relies on the acceleration of \(^3\)He ions by cyclotron waves of protons. This mechanism was studied in detail by Miller et al.\(^1\). They thought that in impulsive type flares there exist a large amount of energetic electrons and when these act on the background plasma, various plasma waves are excited, which are absorbed by the ions following wave-ion resonance, and the ions are accelerated. For instance, the resonance between the electromagnetic cyclotron waves excited by \(\text{H}^+\) ions and \(^3\)He ions can accelerate \(^3\)He ions to the order of MeV/nucleon.

The resonance between the excited shearing Alfven waves and heavy ions can also accelerate the heavy ions to the same order. In the mechanism of two-step acceleration, first there is a preliminary heating \(^8\) (i.e., selective acceleration). The various plasma waves excited by beams of energetic electrons (e.g., cyclotron waves of \(\text{H}^+\) and hybrid waves of \(\text{H-He}\) ions) selectively accelerate \(^3\)He and heavy ions to energies of several tens and hundreds of keV, respectively. When the energy exceeds a certain threshold value, a second acceleration via turbulence and shock waves may further increase the energies to several tens of MeV. Up to the present, the study in this field remains in the stage of stationary solution of the diffusion equation and the fitting of observed energy spectrum \(^8\)–\(^9\), and the questions of the parameters of the acceleration region, the time of acceleration etc. have not been touched upon.

In view of the particle abundance and the characteristics of the spectral distribution of \(^3\)He-rich events, we believe the mechanism of two-step acceleration to be more reasonable \(^5\)–\(^8\). On the basis of the theory of proton acceleration by cascading Alfven waves \(^11\) and the acceleration of Alfven waves by magnetic reconnection during flare outbursts, we assume that a Kolmogorov type of spectral distribution is formed by the cascading of the Alfven waves. By varying the parameters of the Alfven waves and the source region, we obtained numerical solutions of the Fokker-Planck equation, and investigated the time evolution of the spectral distribution of those \(^3\)He and heavy ions that have passed the threshold value after the preliminary heating and been further accelerated by the Alfven waves. We made a comparison with the observational results, which led us to a more thorough understanding of the physical properties of this phenomenon.

2. THEORETICAL MODELS

2.1 Condition of Resonance

According to the dispersion relation \(\omega = k || v_A\) of left-hand Alfven waves propagating in the direction parallel to the magnetic field and according to the condition of wave-particle
resonance \((\omega - s\Omega_i - k_{\parallel}v_{\parallel} = 0)\), one obtains, in the low frequency \((\omega < \Omega_i)\) approximation, the threshold value of particle velocity during wave-particle resonance, \(v_{\parallel} > v_A\), \(v_A\) being the Alfvén velocity, \(\Omega_i\), the cyclotron frequency of the ions, \(S\), the harmonic wave number, \(k_{\parallel}\) and \(v_{\parallel}\), the wave number and particle velocity component parallel to the magnetic field.

Reames et al.\(^\text{[3]}\) analyzed observational data on 228 \(^3\)He-rich events in the period from August 1978 to April 1991 and inferred that the source region of acceleration of \(^3\)He is located in the corona and the parameters of the source region are: \(n_e > 10^{10} \text{ cm}^{-3}\), \(T_e = (3 - 5) \times 10^6 \text{ K}\). Let us suppose that the magnetic field intensity is \(B = 100 \text{ G}\), then the thermal velocity of the ions is \(v_{Ti} = 1.6A^{-1/2} \times 10^7 \text{ cm s}^{-1}\) (where \(A\) is the ion mass), and the Alfvén velocity is \(v_A = 2.18 \times 10^8 \text{ cm s}^{-1}\). It is quite evident that only after a preliminary heating, which makes the ion velocity to be \(v_{\parallel} > v_A\), that a second acceleration by turbulent Alfvén waves may take place.

### 2.2 Fundamental Equations

In the study of the acceleration of protons by cascading Alfvén waves, Miller et al.\(^\text{[11]}\) gave the following Fokker-Planck equation for the evolution of the particle distribution function:

\[
\frac{\partial N(E,t)}{\partial t} = -\frac{\partial}{\partial E} \left[ (A(E,t) - \frac{dE}{dt}|_{\text{loss}})N(E,t) \right] + \frac{\partial^2}{\partial E^2} \left[ D(E,t)N(E,t) \right] - \frac{N(E,t)}{T_{\text{esc}}(E,t)} + Q(E,t).
\]

Here \(N(E,t) = 4\pi p^2 f(p)dp/dE\) is the particle number density, \(D(E,t) = v^2 D(p)\) is the coefficient of diffusion, \(A(E,t)\) is the coefficient of convection, \(T_{\text{esc}}(E,t), Q(E,t)\) and \(dE/dt|_{\text{loss}}\) are, respectively, the particle escape time, the source function of the particles pouring into the source region and the energy loss by Coulomb collisions.

When the frequency of waves \(\omega \ll \Omega_i\) and \(v_{\parallel} \gg v_A\), the diffusion coefficient is\(^\text{[11]}\):

\[
D(p) = 2\pi^2 (ze)^2 \left( \frac{v_A^2}{c} \right)^2 \int_{k_0}^{\infty} dk_{\parallel} \left( 1 - \frac{k_{\parallel}^2}{k^2} \right) W_T(k_{\parallel}) \frac{k^2}{k_{\parallel}^3}.
\]

Here \(k_0 = m_i\Omega_i/p, \Omega_i = zEB/m_ic\), and \(W_T(k_{\parallel}) = I_0k_{\parallel}^{-5/3}\) represents the spectral distribution of the turbulent waves. (We prefer the Kolmogorov spectrum and suppose that it does not change with time in the course of the computation.) In these expressions \(m_i, p, E\) and \(z\) are, respectively, the mass, momentum, energy and number of nuclear charges of the ions, and \(c\) is the velocity of light.

The coefficient of convection \(A(E,t)\) has the following expression\(^\text{[11]}\):

\[
A(E,t) = \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D(p) \frac{\partial E}{\partial p} \right].
\]

Since after preliminary heating the velocity of ions is greater than the thermal velocity of electrons, energy loss term by Coulomb collisions is given by \(^\text{[12]}\):

\[
\frac{dE}{dt}|_{\text{loss}} = \sqrt{\frac{2}{m_i}} \frac{2\pi n_e z^2 e^4 \ln \lambda}{kT_e} \left( \frac{v}{v_e} \right)^2 E^{3/2}.
\]

The escape time of ion diffusion is \(^\text{[11]}\):
\[ T_{\text{esc}} = \frac{9\pi^2 (\sigma \epsilon)^2 L^2}{4 c^2} \frac{1}{p^2 v} \int_{k_0}^{\infty} \frac{1}{k_{||}^2} (1 - \frac{k_{||}^2}{k_0^2}) W_T(k_{||}) dk_{||}. \] (5)

Let us assume that the preliminarily heated ions have a Gaussian type distribution, i.e. \( N(E) = N_0 e^{-10(E - \alpha)^2} \), and the energy density of turbulent Alfvén waves is \( W_T = \alpha B^2 / 8\pi \) (in this work \( \alpha \) is taken to be 0.001-0.005). The minimum wave number \( k_{\text{min}} \) is determined by the scale of magnetic reconnection, \( \alpha \) is the energy of particles after the one-step acceleration.

3. RESULTS OF NUMERICAL COMPUTATION AND THEIR ANALYSIS

After the parameters had been made dimensionless with \( \epsilon = E/E_0 \) (\( E_0 = 1 \text{ MeV} \)) and \( \tau = t\Omega_i \), the discrete form of the equation was numerically solved. Then the parameters of the source region of acceleration and Alfvén turbulence were varied, and we analyzed the internal relation between the process of acceleration and these physical parameters.

(1) We took the background plasma density in the source region of acceleration to be \( n_e = 10^{10} \text{ cm}^{-3} \), the magnetic field strength \( B = 100 \text{ G} \), and the distribution of particles after the one-step acceleration to be \( N(E) = N_0 e^{-10(E - 0.1)^2} \). Then it can be seen from Eqs.(2) and (3) that both the diffusion coefficient and convection coefficient of the waves are proportional to the energy of the waves, that is, the effect of acceleration varies with the wave energy. We only have to have 3 values of the turbulent energy density of Alfvén waves (\( W_T = 0.4, 1, 2 \text{ ergs cm}^{-3} \), \( \alpha = 0.001, 0.0025, 0.005 \)), to obtain an impression of the evolutionary characteristics of the distribution function of \(^3\text{He}\). (See Fig.1.)

![Fig. 1](image-url)

Fig. 1 Evolutionary characteristics of energy spectrum distribution of accelerated \(^3\text{He}\) for 3 different turbulent energy densities

As Fig.1(a) shows, when the energy density of waves is rather small (0.4 ergs cm\(^{-3}\)), it is impossible to accelerate the particles to sufficient energy within a time of the order of seconds, so the spectrum is comparatively soft. When \( W_T = 1 \text{ erg cm}^{-3} \) (Fig.1(b)), the spectral index in the range 0.4-4 MeV/nucleon is 3.5-2.5 after 1-2s of acceleration. When
$W_T$ is 2 ergs cm$^{-3}$ (Fig.1(c), the spectral index is 3.5-2.0 after 0.5-2 s of acceleration. The observed spectral index of $^3$He ions (in the range 0.4-4 MeV/nucleon) is $3.31 \pm 0.27^{[5]}$. This demonstrates that it is possible for Alfvén turbulence to further accelerate, in a few seconds, the $^3$He ions with energies of 100 keV after the one-step acceleration, to the order of MeV. This agrees rather well with the observation.

(2) When we take the background plasma density in the source region of acceleration to be $n_e = 10^9$ cm$^{-3}$, and the turbulent energy density of Alfvén waves $W_T$ to be 0.4 and 2 ergs cm$^{-3}$, and the other parameters the same as in Fig.1, we obtain the curves in Figs.2(a) and (b). Compared to Figs.1(a) and (c), we find that both the energy and time of acceleration, which exhibit the same distribution as $n_e$, diminish together with the decrease of $n_e$. Fig.2(c) shows the case of $n_e = 10^9$ cm$^{-3}$, $B = 50$ G and $W_T = 0.004B^2/8\pi = 0.4$ ergs cm$^{-3}$. By comparing Figs.2(a) and 2(c), we see that in spite of different magnetic field strengths the distributions are alike, when the wave energy density is kept the same. This indicates that it is the energy density that plays the chief role in the acceleration process.

(3) Fig.3 presents the distributions of heavy ions $^{56}$Fe ($Z/A=0.28$) and $^{28}$Mg ($Z/A=0.43$) for the same parameters of the acceleration region and waves as in Fig.1(b). According to the condition of wave-particle resonance, $K_{\|}V_{\|} = \Omega_i$, the smaller the charge-mass ratio of particles, the smaller the corresponding resonance wave number $K_{\|}$, and hence the larger $W_T(K_{\|})$. So the time of acceleration needed for the same distribution becomes shorter. The result of numerical computation agrees with the theoretical expectation. As shown by the observation, for nucleons with energies of 0.4-4 MeV, the spectral index also diminishes, and the spectrum becomes harder with decreasing charge-mass ratio $^{[5]}$. This demonstrates that the theoretical calculation agrees rather well with the observation.

(4) The minimum wave number $K_{\text{min}}$ can be so altered that the energy density of the waves is kept unchanged. Because $I_0 = 0.67W_TK_{\text{min}}^{2/3}$, the smaller the $K_{\text{min}}$ is, the smaller is the $W_T(K_{\|})$ at the corresponding point of resonance, and the longer is the time of acceleration needed for the same distribution. Conversely, a larger $K_{\text{min}}$ means a larger $I_0$, and a larger $W_T(K_{\|})$ at the resonance, and a shorter time to reach a given distribution. Thus our numerical computation agrees with the theoretical analysis. Besides, if the energy

![Image of Figure 2](image-url)
of the one-step accelerated particles is changed to 50 keV or if the initial distribution is changed to $N(E) \propto e^{-(E-\alpha)^2}$ while the other parameters remain the same as in Fig 1(b), then we shall find the curves to remain basically the same. This implies that so long as the threshold condition is satisfied, the influence of the primary distribution of particles on the energy spectrum is not significant.

4. DISCUSSION

(1) When the energy of waves is comparatively small, the equation of two-dimensional momentum diffusion is commonly used to describe the process of turbulent acceleration of particles in magnetized plasma. However, for Alfvén waves Faraday’s law of electromagnetic induction gives rise to $B = n || E$ and $n || = C/v_A > 10$, so magnetic field plays the chief role. If during the wave-particle interaction the effect of scattering is larger than that of acceleration, the distribution may be approximately taken to be isotropic. Therefore, in the above study of acceleration of ions by Alfvén turbulence, we may use the diffusion approximation and solve the one-dimensional Fokker-Planck equation.

(2) This paper gives only the energy spectrum distribution of accelerated $^3$He and heavy ions in the source region of acceleration (located in the solar corona), and does not take into consideration the process of particle transport. What is measured are curves that relate the flux and energy of the particles. If the effect of transport is ignored, we have $\text{flux} \propto N(E)E^{1/2}$. In our next work, we shall consider the evolution of the energy spectrum caused by spatial inhomogeneity in the process of particle transport.

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References