Model of nonlinear kinetic Alfvén waves with dissipation and acceleration of energetic electrons

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The acceleration of energetic electrons has long been one of the most outstanding problems in astrophysics and space physics, and some recent observations from space satellites show that low-frequency electromagnetic fluctuations in the auroral ionosphere and magnetosphere can often be identified as the kinetic Alfvén modes. A model of nonlinear kinetic Alfvén waves is presented here, in which the effect of electron collisional dissipation has been taken into account. The result is a dissipative solitary kinetic Alfvén wave (DSKAW), which can produce a local shocklike structure with a net electric potential drop and which can thereby accelerate efficiently the electrons to the order of the local Alfvén velocity. Since Alfvénic fluctuation is the most common electromagnetic activity in extensive cosmic plasma environments, the present result suggests that the DSKAW could play an important role in the acceleration and energization of cosmic plasmas.

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The production of energetic particles is very common in extensive cosmic plasma environments from auroral electrons in the magnetosphere and energetic particles in solar flares to energetic cosmic rays. The physical mechanism of the acceleration of these energetic particles, however, has long been one of the most outstanding problems in astrophysics and space physics. It is, however, still an open problem as to how the acceleration takes place. On the other hand, intense Alfvénic fluctuation is the most common electromagnetic activity in cosmic plasmas. In particular, some recent studies of in situ observations by space satellites on the polar orbits have shown that the physical nature of the strong electric spikes in the auroral ionosphere and magnetosphere can be explained in terms of a solitary kinetic Alfvén wave (SKAW) because (1) the ratio of the perturbed electric to magnetic fields, \( \Delta E/\Delta B \sim v_A \) (the local Alfvén velocity); (2) the perpendicular scale size \( \lambda_p \sim \) several \( \lambda_e \) (the local electron inertial length); and (3) these spikes are frequently accompanied with strong density fluctuations \( (dn/n \sim 50\%) \) [1–7]. A kinetic Alfvén wave (KAW) can be created when obliquely propagating shear Alfvén waves are affected by the ion gyroradius (for \( \beta > Q \)), or by the electron inertia length (for \( \beta < Q \)) such that a nonzero parallel electric field arises within the wave itself [8,9], and the existence of the SKAWs has been investigated theoretically under various conditions of plasma parameters by many authors [10–14] since the pioneering theoretical work of Hasegawa and Mima in 1976 [15], where \( \beta \) is the ratio of thermal to magnetic pressures, \( Q = m_e/m_i \) is the mass ratio of electrons to ions.

The possibility of linear KAWs accelerating and heating plasmas has been an increasingly interesting topic for discussion extensively in the fields of laboratory [16], space [17], and astrophysical plasmas [18] because of the capability of their parallel electric fields accelerating charged particles as well as the most common Alfvénic fluctuation in cosmic plasmas. However, the possible role of nonlinear KAWs (especially SKAWs) in the accelerating and heating plasmas has not been discussed yet because there is no net electric potential drop over them due to the symmetry in their structures [19]. However, some further analyses of data revealed clearly that electron collisional dissipation could considerably affect the structure and evolution of the SKAWs. For example, on the basis of the analysis for about 100 SKAW events observed by FREJA, Wahlund et al. [20] these events could be classified as three different observational phases, and they were possibly responsible for three different stages in the dynamical evolution of the SKAWs due to the effect of the dissipation caused by the electron collision with turbulent ion acoustic waves [20]. In this paper, we present a model of nonlinear KAWs, in which the effect of electron collisional dissipation will be taken into account. The result is a dissipative solitary kinetic Alfvén wave (DSKAW), which can produce a local shocklike structure with a net electric potential drop and which can thereby accelerate efficiently the electrons to the order of the local Alfvén velocity. In particular, this suggests that the DSKAW could play an important role in the acceleration and energization of cosmic plasmas in the common Alfvénic fluctuations.

The equations governing the dynamics of the DSKAWs propagating in the \( x-z \) plane in a low-\( \beta \) plasma magnetized by a homogeneous ambient magnetic field \( B_0 \) along the \( z \) direction can be written as follows [11–13]:

\[
\begin{align*}
\partial_t n_e + \partial_x (n_e v_{ez}) &= 0, \\
\partial_t n_i + (\omega_i B_0)^{-1} \partial_x (n_i \partial_x E_z) &= 0, \\
(\partial_t + v_{ez} \partial_z) v_{ez} &= -(e/m_e) E_z - v_e v_{ez}, \\
\partial_t E_x - \partial_z E_z &= -\mu_0 n_e v_{ez}, \\
\partial_t B_y &= -\mu_0 n_e v_{ez}, \\
n_e &= n_i = n,
\end{align*}
\]

where \( v_c \) is the collisional frequency of electrons with ions or turbulent waves, \( \partial \) represents the partial derivatives with its subscripts, and the other symbols have their usual mean-
ings. In the above equations we have assumed (i) the low-$\beta$ condition $\beta \ll Q$, so that the thermal pressure may be neglected in Eq. (3), (ii) the low-frequency limitation $\partial_t \ll \omega_{ei}$, so that the $x$ component of the ion velocity may be approximated by the polarization drift velocity in Eq. (2), and (iii) the quasineutrality approximation $n_i = n_e$ in Eq. (6). In addition, the momentum equation of ions has been neglected because of $v_{iz} \sim Q v_{ez} \ll v_{ez}$.

In the wave frame defined in the following form:

$$\eta = k_x x + k_z z - Mt,$$

(7)

taking the unperturbed boundary conditions as

$$\eta \to +\infty \Rightarrow n = n_0, v_{ez} = 0, \partial_q n = 0,$$

(8)

the nonlinear equation governing the density behavior of the DSKAWs can be derived from Eqs. (1)-(6) as

$$d^2 n = 3n^{-1} (d_q n)^2 + k_z^{-2} n^2 (1 - n)(M_z^{-2} - n) + \gamma n d_q n,$$

(9)

and the parallel components of the electron velocity $v_{ez}$ and the perturbed electric field $E_z$ are

$$v_{ez} = -(n^{-1} - 1) M_z$$

(10)

and

$$E_z = k_z M_z^2 [n^{-1} d_q n + \gamma (n^{-1} - 1)],$$

(11)

respectively, where

$$\gamma = \sqrt{\frac{Q}{M \omega_{ei}}}$$

(12)

is the damping coefficient determined by the collisional frequency $\nu_e$, $k_z = \sin \theta$, $k_x = \cos \theta$, $M_z = M/k_z$, and $\theta$ and $M$ are the angles between the wave propagating direction and the ambient magnetic field and the phase speed of the wave in the units of the Alfvén velocity $v_A$, respectively. In the above equations, space, time, density, velocity, and electric and magnetic fields have been normalized to $\lambda_e, \lambda_e/v_A, n_0, v_A, \sqrt{\nu_A B_0}$, and $\sqrt{\nu} B_0$, respectively.

To restore the dimension, the damping coefficient $\gamma$ in Eq. (12) can be rewritten as

$$\gamma = \nu_e/\omega k \lambda_e,$$

(13)

where $\omega = M v_A k \lambda_e$ is the frequency of the wave. For the KAWs in a low-$\beta$ plasma, one has $k \lambda_e \sim O(1)$, and hence $\nu_e/\omega \sim 1$. Neglecting the dissipative effect (i.e., taking $\gamma = 0$), Wu et al. [19] derived an exact analytical solution of Eq. (9), which describes the density distribution of the SKAWs [see Eq. (22) and Fig. 2 in Ref. [19]]. For the case of the DSKAWs with $\gamma \neq 0$, we can obtain numerically the solution of Eq. (9). The solution with the parameters $\gamma = 0.1$, $M_z = 1.29$, and $k_x = \sin 89^\circ$ is illustrated in Fig. 1, where Figs. 1(a)-1(c) are the distributions of the density ($n$), the parallel electric field $E_z$, and the velocity of electrons ($v_{ez}$), respectively. For the sake of comparison, the solution of the corresponding SKAW with the same $M_z = 1.29$ and $k_x = \sin 89^\circ$, but $\gamma = 0$, is presented by the dashed lines in Fig. 1 [19].

From Fig. 1, it is easy to find that the DSKAW produces a local shocklike structure with a scale $\sim 10 \lambda_e$ in the density distribution [see Fig. 1(a)] due to the dissipative effect of $\gamma \neq 0$. In particular, the nonsymmetrical distribution of the electric field [see Fig. 1(b)] can produce a net electric potential drop over the DSKAW. In consequence, the drop can accelerate the electrons to the order of the local Alfvén velocity $v_A$ such that they escape finally by the downstream of the DSKAW [see Fig. 1(c)]. The density jump over the DSKAW, $\Delta n$, and the escaping velocity of the electrons, $v_e$, can be expressed in the parameter $M_z$ as

$$\Delta n = 1 - M_z^{-2}$$

(14)

and

$$v_e = -(M_z^{-2} - 1) v_A = -\Delta n (1 - \Delta n)^{-3/2} v_A,$$

(15)

respectively. Figure 2 plots the escaping velocity $|v_e|$ versus the density jump $\Delta n$, where the dashed line represents the parallel phase speed of the wave $M_z$. For the solution of Fig. 1, one has the density jump $\Delta n = 40\%$ and the escaping velocity $v_e = -0.86 v_A$. 

![Figure 1](https://example.com/figure1.png)

**FIG. 1.** Sketch of the shocklike structure of the DSKAW for the parameters $\gamma = 0.1$, $M_z = 1.29$, and $\theta = 89^\circ$: (a) the density $n$; (b) the parallel electric field $E_z$; and (c) the parallel velocity of electrons $v_{ez}$, where the dashed lines represent the corresponding solution of the SKAW with the same $M_z$ and $\theta$, but $\gamma = 0$ (see Ref. [16]).
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most cases. The result suggests that the DSKAW could play

an important role in the acceleration of the energetic elec-

trons in astrophysical and space plasmas, since the Alfvenic

fluctuation is the most common electromagnetic activity in
cosmic plasmas. As an example, we will discuss below the

possibility of applying the DSKAW to the acceleration of the

Earth’s auroral and solar coronal energetic electrons.

It has long been known that the discrete aurora is associ-

ated with energetic electrons with the energy in the range of

1–10 keV, which impact the ionosphere [21]. The physical

nature of the acceleration of these auroral energetic elec-

trons, however, is still an open problem. It is interesting to

note the fact that the local Alfven velocity \( v_A \) varies along

the auroral magnetic-field lines and reaches its maximum at

the auroral acceleration region that is located in the altitude

\( \sim 5000–12,000 \) km above the auroral ionosphere [7,21].

For the acceleration region, the typical plasma parameters can be

taken as \( B_0=0.06 \) G and \( n_0=10 \) cm\(^{-3}\), and therefore the

local Alfven velocity \( v_A=40,000 \) km/s. Consequently, the

escaping electrons accelerated by the DSKAW from the ac-

celeration region have a typical energy \( \sim m_e v_A^2 \sim 10 \) keV,

which can meet the requirements of the acceleration of the

observed auroral energetic electrons.

On the other hand, it has been widely accepted that the

energetic electrons of a few tens of keV in the solar corona

have been most directly detected through their gyromagnetic

emission in the microwave and their bremsstrahlung

emission in the hard x ray during solar flares [22–25]. Voitenko

[26] demonstrated that the proton beams, which were set up

by the reconnection outflow at the onset of a solar flare,
could excite effectively the KAWs at a high growth rate

(\( \sim 10^5 \) s\(^{-1}\)) and in a short relaxation distance (\( \sim 10 \) km)
in a flare loop. It can be expected that the DSKAW excited by

the flare proton beams can possibly play an important role in

the acceleration of the flare energetic electrons too. In fact,

the typical parameters in the solar flare loops can be taken as

\( B\sim 10^3 \) G, \( n\sim 10^9 \) cm\(^{-3}\), and \( T_e\sim 5\times 10^6 \) K\( \sim 500 \) eV, and

hence the pressure parameter \( \beta\sim 2\times 10^{-5} \sim 0.04Q<<Q \) and

the local Alfven velocity \( v_A\sim 70,000 \) km/s. Consequently,

the escaping electrons accelerated by the DSKAW in the

flare loops have a typical energy of \( \sim m_e v_A^2 \sim 30 \) keV, which

can meet the requirements of the acceleration of the flare

energetic electrons [22–25].

In summary, in this paper we presented a model of non-

linear KAWs, called DSKAW (dissipative solitary kinetic Alfven

wave), in which the effect of the electron collisional dissipation

has been taken into account. The result showed that the

DSKAW produced a local shocklike structure with a net electric

potential drop. In particular, we argue that the

DSKAW can act as an efficient and common acceleration

mechanism of the energetic electrons, and accelerate the

electrons to the order of the local Alfven velocity. The accel-

eration of energetic electrons has been one of the most out-

standing problems in astrophysics, space physics, and plasma

physics for a long time. Considering the fact that the

Alfvenic fluctuation is the most common electromagnetic ac-

tivity in extensive cosmic plasma environments, the present

result suggests that the DSKAW can possibly play an impor-

tant role in the acceleration of cosmic energetic electrons and

in the energization of cosmic plasmas. Finally, as an ex-

ample, we also discuss the possibility of applying the

DSKAW to the acceleration of the Earth’s auroral and solar

flare energetic electrons.

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