A possible explanation for the radio afterglow of GRB 980519: the dense medium effect

X. Y. Wang,1 Z. G. Dai1,2 and T. Lu1,2★
1Department of Astronomy, Nanjing University, Nanjing 210093, China
2LCRHEA, Institute for High-Energy Physics, Chinese Academy of Sciences, Beijing 100039, China

Accepted 2000 April 16. Received 2000 March 15; in original form 1999 December 22

ABSTRACT
GRB 980519 is characterized by its rapidly declining optical and X-ray afterglows. Explanations of this behaviour include models invoking a dense medium environment, which makes the shock wave evolve quickly into the subrelativistic phase, a jet-like outflow, and a wind-shaped circumburst medium environment. Recently, Frail et al. found that the latter two cases are consistent with the radio afterglow of this burst. Here, by considering the transrelativistic shock hydrodynamics, we show that the dense medium model can also account for the radio light curve quite well. The potential virtue of the dense medium model for GRB 980519 is that it implies a smaller angular size of the afterglow, which is essential for interpreting the strong modulation of the radio light curve. Optical extinction arising from the dense medium is not important if the prompt optical–UV flash accompanying the γ-ray emission can destroy dust by sublimation out to an appreciable distance. Comparisons with some other radio afterglows are also discussed.

Key words: hydrodynamics – shock waves – gamma-rays: bursts.

1 INTRODUCTION
In the standard model of gamma-ray bursts (GRBs) (see Piran 1999 for a review), an afterglow is generally believed to be produced by the synchrotron radiation or inverse Compton scattering of the shock-accelerated electrons in an ultrarelativistic shock wave expanding in a homogeneous medium. As more and more ambient matter is swept up, the shock gradually decelerates while the emission from such a shock fades down, dominating at the beginning in X-rays and progressively in the optical to radio energy bands (Mészáros & Rees 1997; Waxman 1997a; Wijers & Galama 1999). In general, the light curves of X-ray and optical afterglows are expected to exhibit power-law decays (i.e. $F \propto t^{-\alpha}$) with the temporal index $\alpha$ in the range 1.1–1.4, given the energy spectral index of electrons $p \sim 2–3$. The observations of the earliest afterglows are in good agreement with this simple model (e.g. Wijers, Rees & Mészáros 1997; Waxman 1997b). However, over the past year, we have come to recognize a class of GRBs, the afterglows of which showed light curve breaks (e.g. GRB 990123; Dai & Lu 1999; Kulkarni et al. 1999a; Harrison et al. 1999). The observations of the earliest afterglows are in good agreement with this simple model (e.g. Wijers, Rees & Mészáros 1997; Waxman 1997b). However, over the past year, we have come to recognize a class of GRBs, the afterglows of which showed light curve breaks (e.g. GRB 990123; Dai & Lu 1999; Kulkarni et al. 1999a; Harrison et al. 1999) or steeper temporal decays (i.e. $F \propto t^{-2}$, e.g. GRB 980519, GRB 980326; Bloom et al. 1999). Explanations for this behaviour include three scenarios.

(1) A jet-like relativistic shock has undergone the transition from the spherical-like phase to a sideways-expansion phase (Rhoads 1999), as suggested by some authors (e.g. Sari, Piran & Halpern 1999; Kulkarni et al. 1999a; Harrison et al. 1999).
(2) The shock wave propagates in a wind-shaped circumburst environment with the number density $n \propto r^{-2}$ (Dai & Lu 1998; Mészáros, Rees & Wijers 1998; Chevalier & Li 1999, 2000; Li & Chevalier 1999).
(3) A dense medium environment ($n \sim 10^{5}–10^{6}$ cm$^{-3}$) makes the shock wave evolve into the subrelativistic phase after a short relativistic one (Dai & Lu 1999a,b).

In the last model, because an afterglow from the shock at the subrelativistic stage decays more rapidly than at the relativistic one, we will expect a light curve break or a long-term steeper decay, depending on the time when it begins to enter into the subrelativistic stage. This scenario has reasonably interpreted the break in the R-band afterglow of GRB 990123 (Dai & Lu 1999a) and the steep decays of the X-ray and optical afterglows of GRB 980519 (Dai & Lu 1999b).

Recently, Frail et al. (1999a) tried to test the first two models (the jet and wind cases) by means of the radio afterglow behaviour of GRB 980519 and found that the wind model described it rather well. Owing to the strong modulation of the light curve, however, they could not draw a decisive conclusion for the jet case. In this paper, we will examine the possibility of describing the evolution of the radio afterglow in terms of the dense medium model. As this scenario involves the transition phase of the shock wave from the relativistic stage to the subrelativistic, we have considered the transrelativistic shock hydrodynamics in the numerical study. We present the asymptotic result of the fitting of the radio data in...
Section 2, and then the numerical result in Section 3. In Section 4, we show that the optical extinction arising from the dense circum-burst medium is not important, because the prompt optical–UV radiation, caused by the reverse shock, can destroy the dust by sublimation out to a substantial distance, as proposed by Waxman & Draine (1999). Finally, we give our discussions and conclusions.

2 ASYMPTOTIC BEHAVIOUR OF THE RADIO AFTERGLOW IN THE SUBRELATIVISTIC STAGE

GRB 980519 is the second-brightest GRB in the BeppoSAX sample. Its optical afterglow measured since \( -8.5 \) h after the burst exhibited rapid fading, consistent with \( r^{-2.05 \pm 0.04} \) in BVRI (Halpern et al. 1999; Djorgovski et al. 1998), and the power-law decay slope of the X-ray afterglow, \( \alpha_X = 2.07 \pm 0.11 \) (Owens et al. 1998), was in agreement with the optical. The spectrum in the optical band alone is well fitted by a power-law \( \nu^{-1.20 \pm 0.25} \), while the optical to X-ray spectrum can also be fitted by a single power-law \( \nu^{-1.05 \pm 0.10} \). The radio emission was observed with the Very Large Array (VLA) (Frail et al. 1999a) from about 7.2 h after the burst, and referred to as VLA J232221.5+771543. The radio light curve shows a gradual rise to a plateau followed by a decline until below detectability after about 60 d. There are some large variations in this data, which are believed to be caused by interstellar scattering and scintillation (ISS; Frail et al. 1999a).

As discussed by Dai & Lu (1999b), the steep decays of the X-ray and optical afterglows of GRB 980519 can be attributed to the shock evolution into the subrelativistic phase 8 h after the burst as the result of the dense circum-burst medium. During such a subrelativistic expansion phase, the hydrodynamics of the shocked shell is described by the self-similar Sedov–von Neumann–Taylor solution. The shell radius and its velocity scale with time as \( r \sim t^{2/5} / \beta \) and \( \beta \sim t^{-3/5} \), where \( t_{10} \) denotes the time measured in the observer frame. Then, we obtain the synchrotron peak frequency \( f_{np} \propto t_{10}^{-3} \), the cooling frequency \( f_c \propto t_{10}^{-1} \), the peak flux \( F_{np} \propto t_{10}^{3/5} \) and the self-absorption frequency \( f_a \propto t_{10}^{1/5} / \nu_2^{2/5} \), the observed frequency \( f_2 \) at the peak flux \( F_{np} \), the peak frequency \( f_{np} \), and the self-absorption frequency \( f_a \) are given by

\[
F_2 = \begin{cases} (f_2 / f_{np})^{-(p-1)/(p+2)} (f_a / f_c) F_c \propto \nu^{p-2} t_{10}^{1/10} & \text{if } f_{np} < \nu < f_c; \\
(f_2 / f_{np})^{-(p-1)/2} F_c \propto \nu^{-(p+1)/2} (p^{21} - 15p^{10}) / \nu_n^{10} & \text{if } \nu < \nu_c; \\
(f_2 / f_{np})^{-(p-1)/2} (f_a / f_c) F_c \propto \nu^{p-2} t_{10}^{4-3p}/2 & \text{if } \nu > f_a. 
\end{cases}
\]

If the observed optical afterglow was emitted by slow-cooling electrons and the X-ray afterglow from fast-cooling electrons, and if \( p = 2.8 \) then according to equation (1), the decay index \( \alpha_R = (21 - 15p)/10 = -2.1 \) and \( \alpha_X = (4 - 3p)/2 \approx -2.2 \), in excellent agreement with observations. Also, the model spectral index at the optical to X-ray band, \( \beta = -(p-1)/2 \approx -0.9 \), is quite consistent with the observed one \( -1.05 \pm 0.10 \). Furthermore, from the information regarding X-ray and optical afterglows, Dai & Lu (1999b) have inferred the physical parameters of this burst as follows:

\[
\begin{align*}
E & \sim 3 \times 10^{52} \text{ erg}, \\
\epsilon_e & \sim 0.16, \\
\epsilon_B & \sim 2.8 \times 10^{-4}, \\
n & \sim 3 \times 10^5 \text{ cm}^{-3}, \\
z & \sim 0.55.
\end{align*}
\]

3 TRANSRELATIVISTIC SHOCK HYDRODYNAMICS, SELF-ABSORPTION EFFECT AND RADIO DATA FITTING

We consider an instantaneous release of a large amount of energy \( E \) in a constant density external medium. The energy released in the medium drives a shock wave, the dynamic evolution of which from the relativistic to the subrelativistic phase can be described approximately in the following way.

Let \( r \) be the shock radius, \( \gamma \) and \( \Gamma \) be, respectively, the Lorentz factors of the shell and the shock front, and \( \beta \) be the velocity of the shock front. As usual, the shock expansion is assumed to be adiabatic, during which the energy is conserved, and we have (Blandford & McKee 1976)

\[
\frac{4}{3} \pi \sigma m \gamma_0 r^2 \sim c^2 = E,
\]

where \( \sigma \) is a coefficient: \( \sigma \approx 0.35 \) when \( \beta \to 1 \) and \( \sigma \approx 0.73 \) when \( \beta \to 0 \). As in Huang, Dai & Lu (1998), we use an approximate expression for \( \sigma \): \( \sigma = 0.73-0.38 \beta \).

The radius of the shock wave evolves as (Huang et al. 1998)

\[
\frac{dr}{dt} = Bc(\gamma + \sqrt{\gamma^2 - 1})/(1 + z),
\]

where \( E \) is shock energy, \( z \) is the redshift of the burst and \( \epsilon_e \) and \( \epsilon_B \) are the electron and magnetic energy fractions of the shocked medium, respectively.

After the 60-d radio observational data being published, we promptly checked the dense medium model, and found that the asymptotic analysis can approximately describe the radio behaviour. The analysis is as follows: adopting the inferred values of the physical parameters in equation (2), the detected frequency \( v_h = 8.46 \text{ GHz} \) equals \( v_a \) at about day 12; thus, according to equation (1), we expect that before this time the radio emissions rise as \( t_{12}^{3/5} \) and then decay as \( t_{12}^{1/10} \) after the self-absorption frequency \( v_a \) falls below \( v_h \). This simple asymptotic solution agrees qualitatively with observations, as showed in Fig. 1 by the dotted line. This preliminary analysis stimulated us to fit the radio data with a more detailed model by taking into account the transrelativistic shock hydrodynamics and the strict self-absorption effects of the synchrotron radiation.
and the jump conditions of the shock are given by (Blandford & McKee 1976)
\[ n' = \frac{\hat{\gamma}y + 1}{\gamma - 1}, \quad e' = \frac{\hat{\gamma}y + 1}{\hat{\gamma} - 1}(\gamma - 1)nmc^2, \]
(5)
\[ \Gamma^2 = \frac{(\gamma + 1)\hat{\gamma}(\gamma - 1) + 1}{\gamma(2\hat{\gamma} - 1)} + 1, \]
(6)
where \( e' \) and \( n' \) are the energy and the number densities of the shell in its comoving frame and \( \hat{\gamma} \) is the adiabatic index, which equals 4/3 for ultrarelativistic shocks and 5/3 for subrelativistic shocks. A simple interpolation between these two limits \( \hat{\gamma} = (4\gamma + 1)/(3\gamma) \) gives a valid approximation for transrelativistic shocks (Dai, Huang & Lu 1999). Using the above equations, we can now obtain numerically the evolution of \( r(t_{0}) \) and \( \gamma(t_{0}) \) in the transrelativistic stage, given proper initial conditions.

As usual, we assume that the distribution of relativistic electrons with the Lorentz factor \( \gamma_e \) takes a power-law form with the number density given by \( n(\gamma_e) \propto C\gamma_e^{-\beta} \) above a low limit \( \gamma_{\text{min}} \), which is determined by the shock velocity,
\[ \gamma_{\text{min}} = \frac{(p - 2) m_0}{(p - 1) m_e}(\gamma - 1). \]
Also, the energy densities of electrons and magnetic fields are assumed to be proportional to the total energy density \( e' \) in the comoving frame as \( U'_e = \epsilon_e e' \) and \( B'_e = (8\pi\epsilon_B e')^{1/2} \). Thus, from the standard theory of the synchrotron radiation (Rybicki & Lightman 1979; Li & Chevalier 1999), we have expressions for the effective optical depth and the self-absorbed flux:
\[ \tau_e = \frac{3\gamma_e^{1/2} m_0 g B'_e}{4 \pi mc^2} \left( \frac{4\pi mc^2}{3q} \right)^{\nu/2} F_{\nu}(\nu/\nu_m) CB_{\nu/2}(\gamma/\nu_m)\Delta \nu', \]
(7)
\[ \nu_m = \frac{3\gamma_e^{1/2} m_0 g B'_e}{4 \pi mc^2}, \quad C = (p - 1)n' \gamma_{\text{min}}^{-1}, \quad \Delta \nu' = r/\eta, \]
(8)
\[ F_{\nu} = (1 + z)D_{L}\pi \left( \frac{r}{D_{L}} \right)^{2/3} \left( \frac{4\pi mc^2}{3qB'_e} \right)^{1/2} \times \frac{F_{\nu}(\nu/\nu_m)}{F_{\nu}(\nu/\nu_m)}(1 - e^{-\tau_e}), \]
(9)
where \( F_{\nu}(x) \) and \( F_{\nu}(x) \) are defined by equation (5) in Li & Chevalier (1999), \( m \) and \( q \) denote the mass and charge of the electron, \( D_L \) is the luminosity distance of the burst, assuming a flat Friedman universe with \( H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( \eta \sim 10 \), characterizing the width of the shock shell. Here \( D_L = 1/(\gamma(1 - \beta)) \) describes the relativistic effect, and \( \nu \) is related to the corresponding frequency \( \nu' \) in the comoving frame by \( \nu = D\nu'/(1 + z) \).

Using the above full set of equations, we computed the radio flux at the frequency \( \nu_m = 8.46 \text{ GHz} \) and plotted the model fit in Fig. 1 as the solid line. We find that the following combination of the parameters fits almost all valid data rather well: \( E \sim 0.8 \times 10^{52} \text{ erg}, \quad \epsilon_e \sim 0.2, \quad \epsilon_B \sim 1 \times 10^{-4}, \quad n \sim 1 \times 10^{2} \text{ cm}^{-3}, \quad \eta \sim 0.55 \) and the Lorentz factor at the initial time \( t_0 = 1/3 \text{ d} \) and \( \gamma = 1.2 \) (\( \beta \sim 0.55 \)). We stress that these parameters are in excellent agreement with those inferred independently from the X-ray and optical afterglows (Dai & Lu 1999b), as listed in equation (2).

Clearly, there are some large-amplitude variations in the observed light curve (e.g. about 18 d after the burst), which is believed to be caused by diffusive scintillation. Also plotted in Fig. 1 is the fit (dashed line) computed with the subrelativistic model as presented in the appendix of Frail, Waxman & Kulkarni (1999c). The fit with this model was obtained by adopting the initial conditions \( t_0 = 1/3 \text{ d} \) and \( r_0 = 1.6 \times 10^{16} \text{ cm} \) and the parameter values (\( \epsilon_c, \epsilon_p, E, \gamma, \alpha \)) the same as those in Dai & Lu (1999b). Comparing the transrelativistic model with the subrelativistic one, we can easily see that the relativistic effect (as characterized by \( D = 1/(\gamma(1 - \beta)) \) in equation 9) flattens the rising phase at earlier times, making the transrelativistic model agree better with the observations, while at later times both the fitting curves tend towards the asymptotic solution (i.e. \( F_{\nu} \propto t_{0}^{-2} \)).

### 4 DUST SUBLIMATION AND OPTICAL EXTINCTION BY A DENSE MEDIUM

One may ask whether the dense circumburst medium may cause large extinction in the optical afterglow of GRB 980519. A crude estimate is as follows. At the time that the blast wave transits to the subrelativistic stage \( (t_0 \sim 1/3 \text{ d}, \beta \sim 0.55, \gamma \sim 1.2) \), from equation (3) we derived its radius to be \( r \sim 2.1 \times 10^{16} \text{ cm} \). Therefore the characteristic column density through the medium into which the blast wave is expanding is about \( n\tau = 2.1 \times 10^{23} \text{ cm}^{-2} \) with a corresponding \( A_V \) of 1.3 mag in the rest frame of the absorber. This column density is comparable to (but slightly larger than) the Galactic 21-cm column density \( (1.74 \times 10^{21} \text{ cm}^{-2}; \text{Halpern et al. } 1999) \) in the direction of GRB 980519. If \( A_V \) scales linearly with \( 1/\Lambda \), the absorption in our observed B band for this optical transient is about 1.3\((1 + z) \sim 2 \text{ mag. twice the value adopted by Halpern et al. (1999)} \) for correction of the relative extinction.

In the above estimate, we have made a questionable assumption, i.e. the dense medium around the burst has a standard gas-to-dust ratio. However, this may be not realistic, considering that the dust around the burst can be destroyed because of sublimation out to an appreciable distance (\( \sim \) a few pc) by the prompt optical–UV flash (Waxman & Draine 1999; hereafter WG99) accompanying the prompt burst. Below we will give an estimation of the destruction radius for the dense medium case, following WG99.

The prompt optical flash detected accompanying GRB 990123 (Akerlof et al. 1999) suggests that, at least for some GRBs, \( \gamma \)-ray emission is accompanied by prompt optical–UV radiation with luminosity in the 1–7.5 eV range of the order of \( 10^{50}(\Delta \Omega/4\pi) \text{ erg s}^{-1} \) for typical GRB parameters, where \( \Delta \Omega \) is the solid angle into which \( \gamma \)-ray and optical–UV emission is beamed (WG99). The most natural explanation of this flash is emission from a reverse shock propagating into the fireball ejecta shortly after it interacts with the surrounding gas (Sari & Piran 1999; Mészáros & Rees 1999). As for GRB 980519, with the parameter values as \( \epsilon_e \sim 0.2, \epsilon_B \sim 10^{-4}, \quad n \sim 10^{5} \text{ cm}^{-3}, \quad E \sim 8 \times 10^{51} \text{ erg} \) and the burst duration \( \Delta t \sim 70 \text{ s} \), we derive the luminosity in the 1–7.5 eV range to be about \( L_{1-7.5} \sim 5 \times 10^{48} \text{ erg s}^{-1} \). (Here we have assumed that electron and magnetic field energy fractions in the reverse shock are similar to those in the forward shock: Wang, Dai & Lu 1999b). The condition for the grain to be completely sublimed during the prompt flash time is
\[ T > T_c = 2300 \text{ K} \left[ 1 + 0.033 \ln \left( \frac{a-5}{\Delta t/10 \text{ s}} \right) \right], \]
(10)
where \( T \) is the grain temperature, determined by equation (8) of WG99, and \( a = a-5 \times 10^{-5} \text{ cm} \) is the radius of the dust grain. Then, according to equation (17) of WG99, the radius out to which the prompt flash can heat grains to the critical temperature
Frail et al. (1999c) inferred that the fireball has undergone a transition to subrelativistic expansion at $t \sim 100$ d, consistent with the inferred low ambient density $n \sim 1$ cm$^{-3}$ (but also see Chevalier & Li 1999a). On the other hand, some radio afterglows (e.g. GRB 990510, GRB 981226; Frail et al. 1999b) show similar behaviour to GRB 980519, that is they exhibit a slow rise to the maximum for a relatively short time and then a fast decline until they are below detectability. It is likely that the shocks of these bursts entered into the subrelativistic stage after a short relativistic one, and our above model can also describe their radio afterglows. Harrison et al. (1999) had interpreted the broad-band lightcurve break in the afterglows of GRB 990510 as arising from a jet-like outflow. We speculate that another possible explanation is that the shock had entered into the subrelativistic stage after $\sim$1 d as a result of the combination of the dense medium and jet effects (Wang et al., in preparation), the latter of which may be real, in consideration of the large inferred isotropic energy. The radio afterglow of GRB 990123 is unique for its ‘flare’ behaviour (Kulkarni et al. 1999b), the most natural explanation for which is that it arises from the reverse shock, as evidenced by the prompt optical flash (Sari & Piran 1999). Our preliminary computation (using the transrelativistic model) shows that the radio emission from the forward shock in the dense medium model is significantly lower than that from the reverse shock and declines quickly after the peak time, if a jet-like outflow with an opening angle $\theta \sim 0.2$, as required by the ‘energy crisis’ of this burst, is invoked (Wang et al., in preparation). Moreover, the fast decline of the radio emission from the forward shock, which is caused by the deceleration of the shock in the subrelativistic stage, can be consistent with the non-detection even 3 d after the burst.

In summary, we argue that the dense medium model, which has interpreted the optical to X-ray afterglows of GRB 980519 quite well, can also account for the radio afterglow excellently. The circumburst environment can affect the evolution of GRB afterglows significantly (Mészáros et al. 1998; Panaitescu, Meszaros & Rees 1998; Wang et al. 1999a). For a low ($n \sim 1$ cm$^{-3}$), homogeneous density environment, the shock waves stay at the relativistic shock stage for quite a long time, while for the dense medium case, the shock wave quickly enters into the subrelativistic stage. Recently, a generic dynamic model for the evolution of shocks from the ultrarelativistic phase to the subrelativistic one has also been developed by Huang, Dai & Lu (1999). The afterglow of the optically thin radiation (e.g. optical and X-rays) from the shock at the subrelativistic stage decays more rapidly than at the relativistic one. As for the radio afterglow (usually $v_{\text{so}} < v_s$), the dense medium model predicts a slow rise ($v_0 < v_s$), followed by a round peak and a late steep decline ($v_0 > v_s$), tending towards the behaviour of the optical and X-ray afterglows. Clearly, this behaviour is different from the jet model in the early epoch. It is, however, somewhat similar to the wind model, making it difficult to distinguish between them through the radio observations.

**ACKNOWLEDGMENTS**

We thank the referee, Dr R. Wijers, for his valuable suggestions and improvements on this manuscript. XYW also thanks Dr Y. F. Huang for helpful discussions. This work was supported by the National Natural Science Foundation of China under grants 19773007 and 19825109 and the Foundation of the Ministry of Education of China.
REFERENCES

Akerlof C. W. et al., 1999, Nat, 398, 400
Blandford R. D., McKEE C. F., 1976, Phys. Fluids, 19, 1130
Bloom J. S. et al., 1999, Nat, 401, 453
Kulkarni S. R. et al., 1999a, Nat, 398, 389

This paper has been typeset from a \LaTeX\ file prepared by the author.