Restoration of Space Object Images by Using A Maximum Entropy Method† ⋆

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Abstract Due to the various factors of image degradation in the optical observation of space objects, the imaging process is affected, it reduces the detection accuracy and brings difficulty to the high-precision positioning calculation. In order to improve the positioning accuracy of space objects, the maximum entropy method is adopted for image restoration, with an a priori PSF (Point Spread Function) model. The measuring errors of object positions before and after restoration are compared to investigate the effectiveness of the image restoration. The experimental results indicate that the influences of image degradation are reduced, and the positioning accuracy of degraded images is obviously improved by using the maximum entropy method.

Key words astrometry, techniques: image processing, space vehicles

1. INTRODUCTION

The ground-based optical observation is an effective measure for probing space objects. To make high-precision monitoring of space objects by using ground-based optical telescopes is of great significance for the recognition of spatial attitude, the computation of collision forewarning, and the safety insurance of orbiting space vehicles¹. In observations, the telescope follows the motion of an object according to the corresponding observation strategy², because of the rather high angular velocity of the object relative the observational station, the tracking errors increase because of the mechanical instability of the telescope in its rapid motion; meanwhile, the image profiles on the observed CCD images are undersaturated,

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even undersampled because of the relatively short exposure time\textsuperscript{[3]}; moreover, in order to adapt to the observation of space objects, the multi-channel readout and the stripped front mechanical shutter that generally adopted by the CCD cameras cause also extra image contaminations\textsuperscript{[4–5]}, it is, therefore, difficult to detect and precisely locate the object images, when they are positioned on the edges of image blocks of different readout channels or near the smears of bright stars. So, it is necessary to consider all these factors, which will degrade the observed images, reduce the imaging quality, affect the data processing, and lower the observing accuracy of space objects.

Along with the development of computer techniques, the digital image processing technique has got more and more applications. An important branch of the image processing is the image restoration, which constructs an image degradation model according to the image degradation process or phenomenon, and restores the images by choosing a proper restoration algorithm on the basis of this model, so that the original images may be restored and the image quality can be improved\textsuperscript{[6–7]}. The image restoration has been widely applied to various astronomical fields\textsuperscript{[8]}, such as the reconstruction of the images observed by IRAS (Infrared Astronomical Satellite)\textsuperscript{[9]}, the mid-infrared image processing for revealing the interior structure of the active galactic nucleus NGC 1068\textsuperscript{[10]}, and the elimination of telescope’s tracking errors\textsuperscript{[11]}, and so on. In this paper, an image restoration method is adopted on the basis of the maximum entropy estimation to treat the observed images of space objects in order to reduce the influences of image degradation on the data processing, and to raise the observing accuracy.

In this paper, firstly, the basic theory about the image degradation and the basic principle and procedures of the maximum entropy method for restoration are briefly presented, and then multiple frames of seriously degraded GPS satellite images are treated, and the observing accuracy of the satellite is obtained by a comparison between the measured satellite positions on the images and the interpolated values of the precise ephemeris. Finally, the result of image restoration is analyzed and discussed by means of a comparison between the observing accuracies of the satellite before and after the image restoration.

\section{2. BASIC PRINCIPLE AND METHOD}

For two-dimensional images, let $O(x,y)$ be the non-degraded true value, and $I(x,y)$, the collected degraded image. We assume that the imaging process is linear in the space domain, and satisfies the requirement of translational invariability. Then it can be expressed as

\begin{equation}
I(x,y) = \int_{x_1=-\infty}^{+\infty} \int_{y_1=-\infty}^{+\infty} P(x-x_1,y-y_1)O(x_1,y_1)dx_1dy_1 + N(x,y)
= O(x,y) \otimes P(x,y) + N(x,y),
\end{equation}

in which $P(x,y)$ is the general imaging function, $N(x,y)$, noise in the imaging process, $\otimes$ indicates a convolution operation. It must be pointed out that $P(x,y)$ represents the
influence of the whole imaging process on the original image, which may be further divided into some parts as follows\cite{12}:

\[ P(x,y) = A(x,y) \otimes H(x,y) \otimes \Pi(x,y), \quad (2) \]

in which \( A(x,y) \) indicates the influence of atmospheric seeing on the imaging process, \( H(x,y) \), that of the telescope’s optical system, \( \Pi(x,y) \), the CCD pixel sampling function in the space domain. According to the relevant properties of the Fourier transform, the convolution in the space domain can be transformed into a product in the frequency domain, the equation (1), therefore, can be expressed as follows:

\[ \hat{I}(u,v) = \hat{O}(u,v) \times \hat{P}(u,v) + \hat{N}(u,v). \quad (3) \]

The image restoration is, with the known \( I(x,y) \) and the a priori information of both \( P(x,y) \) and \( N(x,y) \), to estimate the values of the original image, which should be optimal according to the corresponding operation criterion. In practice, the method is classified into two types, one is linear, which covers the inverse filter restoration, Wiener filter restoration, etc.; another, non-linear, which covers the Bayesian estimation restoration, maximum entropy restoration, etc..

According to Shannon’s definition\cite{13–14}, the entropy is a kind of measurement index for the average amount of information contained in a signal, it describes the disorder degree of a system and increases with its raising stability. By virtue of the maximum entropy method the quality of an image is expressed with the entropy, of which the value indicates the amount of information. The larger the entropy of an image, the more the information, and the better the image quality. The signal entropy and noise entropy characterize the smoothness and homogeneity of an image, so the optimal criterion in the maximum entropy method for restoration is the maximum of both the signal entropy and noise entropy after the restoration. It is an ideal method of image restoration. Under the assumption that the image function is non-negative, namely,

\[ O(x,y) \geq 0, \quad (4) \]

which is tenable for the gray CCD images of observed space objects, the total energy of a single frame of image is defined as

\[ E = \sum_x \sum_y O(x,y). \quad (5) \]

the signal entropy of an image is defined by Frieden as

\[ H_s = - \sum_x \sum_y O(x,y) \ln O(x,y), \quad (6) \]

and the noise entropy is defined as

\[ H_N = - \sum_x \sum_y [N(x,y) + B] \ln[N(x,y) + B], \quad (7) \]
in which $B$ is the minimum absolute value of noise. In order to make the signal entropy and noise entropy after restoration become maximal under the constraint of an image degradation model, the Lagrangian function is introduced:

$$
R = \rho H_N + \sum_{x=1}^{k} \sum_{y=1}^{k} \lambda_{mn} \left\{ \sum_{x=1}^{k} \sum_{y=1}^{k} P(m-x,n-y)O(x,y) + N(m,n) - B - I(m,n) \right\} + \\
\beta \left\{ \sum_{x=1}^{k} \sum_{y=1}^{k} O(x,y) - E \right\},
$$

(8)
in which $\lambda_{mn}$ and $\beta$ are Lagrangian multipliers, $k$, pixel number of the image to be treated, $\rho$, weighing factor used for emphasizing the relation of interaction between $H_f$ and $H_N$. The following extreme-value conditions

$$
\frac{\partial R}{\partial O(x,y)} = 0, \\
\frac{\partial R}{\partial N(x,y)} = 0
$$

(9)
are used to obtain a set of equations about $O(x,y)$ and $N(x,y)$, which can be solved by an iterative procedure\cite{15}.

3. EXPERIMENTAL METHOD

In our experiment, the data of GPS satellite optical observations seriously affected by the image degradation are chosen to analyze the effect of the maximum entropy method of restoration on the object detection. It must be specified that the GPS satellites are the typical space objects, whose images have the typical observational characteristics of space objects. The conclusion obtained from the analysis of the effect of image restoration method on the observing accuracy of GPS satellites is of generality; meanwhile, the high-precision satellite positions at the observing instants, which may be taken as reference values, can be obtained by interpolating the precise ephemeris of GPS satellites, it is advantageous to the analysis of calculation accuracies obtained by various methods of treatment. Some pieces of information concerning the collected images and the telescope to be used are listed in Table-1.
Table 1 Information of images

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>$2048 \times 2048$</td>
</tr>
<tr>
<td>Field of view</td>
<td>$4.4^\circ \times 4.4^\circ$</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>$7.73''$</td>
</tr>
<tr>
<td>Exposure time</td>
<td>2500 ms</td>
</tr>
<tr>
<td>CCD operating mode</td>
<td>Full frame</td>
</tr>
<tr>
<td>Number of channels</td>
<td>4</td>
</tr>
</tbody>
</table>

In the experiment, 99 frames of original images on 2 arcs have been collected. During the observation on the satellite, the telescope follows the object’s motion and images the object at the center of viewing field. Owing to the relative motion, star images show themselves like strips, while the object image, a point. Because that the camera’s front shutter has been stripped off, smears are generated by bright stars on the whole CCD image; at the same time the image is readout from 4 channels, the background reading of each image block differs; in addition to the effects of other image degradation factors, the positioning accuracy of the object is reduced. By interpolating the high-precision ephemeris of the GPS satellite for the observing instants, we can obtain the precise positions of the object at these instants, which are converted into the topocentric coordinates of J2000.0 mean epoch, so that they may be taken as the real values of object positions in the same coordinate system as that in which the astronomical measurements have been made. A comparison between the astronomical positioning results and the interpolated values of the precise ephemeris gives rise to residuals, which characterize the accuracy of observation.

To test the effectiveness of the maximum entropy method for image restoration and its advantage for improving the positioning accuracy of the object, this method is used to process the collected CCD images. It is difficult in our practical calculation to establish a degradation model $P(x,y)$ of the telescope system by modelling the CCD images, owing to the effect of image degradation and the brightness difference between two different frames of images$^{[16]}$. A simple procedure, therefore, is adopted as follows: firstly, the full widths at half maximum (FWHM) of satellite images on all the CCD images are measured, and the distributions of the full widths at half maximum of object images on the two arcs are shown in Figure 1.

On Arc 1, the statistical result of FWHMs of object images is $(2.99 \pm 0.53)$ pixel, while on Arc 2, that is $(2.99 \pm 0.20)$ pixel. A point spread function (PSF) model, whose FWHM value is a little less than that of object images, is adopted to make the scattered object images more concentrated and the edges of object images more sharpened, in consideration of the effect of image degradation on the FWHMs of object images, and the reconstruction of image profiles as the motive of image restoration. In our experiment, taking the calculation time of image restoration into account, a $5 \times 5$ two-dimensional Gauss point spread function
model with FWHM = 2.5 pixel is selected, which is regarded as \( P(x, y) \) to join in the image restoration procedure. The model adopted is as follows:

\[
\text{PSF}_{\text{gauss}} = \begin{pmatrix}
0.034673 & 0.119131 & 0.179633 & 0.119131 & 0.034673 \\
0.119131 & 0.409323 & 0.617200 & 0.409323 & 0.119131 \\
0.179633 & 0.617200 & 0.930649 & 0.617200 & 0.179633 \\
0.119131 & 0.409323 & 0.617200 & 0.409323 & 0.119131 \\
0.034673 & 0.119131 & 0.179633 & 0.119131 & 0.034673
\end{pmatrix}.
\] (10)

As far as the original images and the restored ones are concerned, the Sextractor software \(^{17}\) is used to obtain the coordinates of the object image and those of star images on the CCD image, which are applied to the astronomical positioning of images \(^{18-19}\) to acquire the measured equatorial coordinates \((\alpha_o, \delta_o)\) of the object. They are in turn applied to the comparison with \((\alpha_c, \delta_c)\), the reference values interpolated from the precise ephemeris, to obtain the corresponding measuring errors \((\sigma_\alpha, \sigma_\delta)\) as calculated as follows:

\[
\begin{align*}
\sigma_\alpha &= (\alpha_o - \alpha_c) \times \cos \delta_c, \\
\sigma_\delta &= \delta_o - \delta_c.
\end{align*}
\] (11)

For the multiple frames of images on the same arc the root mean square value (RMS) of measuring errors of the object positions on the whole arc is calculated with outliers rejected to indicate the final precision of object positions on this arc. The calculation procedure for the observational arc with the number \(L\) of data points is given below:

\[
\begin{align*}
\text{RMS}_\alpha &= \sqrt{\frac{\sum (\alpha_o - \alpha_c)^2 \times \cos \delta_c^2}{L-1}}, \\
\text{RMS}_\delta &= \sqrt{\frac{\sum (\delta_o - \delta_c)^2}{L-1}}.
\end{align*}
\] (12)
Fig. 2 Contour maps of the 8 couples of object images before ((a)∼(h)) and after ((i)∼(p)) restoration
4. DISCUSSION

Eight typical instances of seriously degraded GPS satellite images are chosen, and the contour maps of its original images and those restored with the maximum entropy method are shown in Figure 2. It can be found from their result of treatment that after the restoration the image profiles are more regular, the noises around an image are suppressed to a certain extent by the image restoration method, and the image shapes are reconstructed. The positioning accuracies of the 8 couples of images are listed in Table 2.

<table>
<thead>
<tr>
<th>Number</th>
<th>Deviation before restoration/&quot;</th>
<th>Deviation after restoration/&quot;</th>
<th>Number of background stars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.33 -19.34</td>
<td>-0.27 1.17</td>
<td>287</td>
</tr>
<tr>
<td>2</td>
<td>19.11 -14.81</td>
<td>-0.31 -1.90</td>
<td>271</td>
</tr>
<tr>
<td>3</td>
<td>21.31 -15.95</td>
<td>-2.02 -1.10</td>
<td>262</td>
</tr>
<tr>
<td>4</td>
<td>37.30 -15.58</td>
<td>0.42 -1.21</td>
<td>301</td>
</tr>
<tr>
<td>5</td>
<td>40.78 6.45</td>
<td>3.46 0.15</td>
<td>354</td>
</tr>
<tr>
<td>6</td>
<td>26.94 8.62</td>
<td>1.77 2.00</td>
<td>337</td>
</tr>
<tr>
<td>7</td>
<td>27.04 -4.61</td>
<td>1.08 1.28</td>
<td>313</td>
</tr>
<tr>
<td>8</td>
<td>41.32 -7.59</td>
<td>3.85 1.27</td>
<td>363</td>
</tr>
</tbody>
</table>

It can be seen from the table that thank of the relatively wide viewing field of images, there are about 300 calibration stars, and the background stars applied to the astronomical positioning are quite sufficient. According to the theory of Pascu et al.[20], when there are sufficient background stars, the astronomical positioning accuracy of the object depends on the centering accuracy of the object image. That the original images are affected by the image degradation results in rather great errors in the measurements of mass center, which may come to the magnitude of 10". Obviously, it is unacceptable for practical engineering applications. After the image processing, the positioning accuracy of images, however, increases substantially to attain an ideal state, namely, the influences of image degradation are considerably reduced, as a result, the measuring accuracies of right ascensions and declinations are generally better than a half pixel, in which the best can reach 0.15", in contrast to 7.73", the pixel resolution of the facility.

For the two experimental arcs, the measured residuals of all the data points before and after restoration are shown in Figures 3∼4, and the observational results of the two arcs with the outliers greater than 3σ rejected are listed in Table 3. It is clear from the table that the measuring accuracy of original images are relatively low, even if the data points greater than 3σ have been rejected. The maximal deviation of measured values of image centers of mass from the reference values is as large as 3 pixel. After image restoration, the measuring accuracies of object images are greatly improved. With the maximum entropy method used for image restoration, the effect of image degradation is dispelled, and the
positioning accuracies of seriously degraded images are amazingly raised.

Table 3  Related information of measurements for two arcs (Unit: ″)

<table>
<thead>
<tr>
<th>Arc ID</th>
<th>Before restoration</th>
<th>After restoration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS of RA</td>
<td>RMS of Dec</td>
</tr>
<tr>
<td>1</td>
<td>12.91</td>
<td>8.05</td>
</tr>
<tr>
<td>2</td>
<td>24.01</td>
<td>3.88</td>
</tr>
</tbody>
</table>

Fig. 3  Residual error distributions of object positions of Arc 1

Fig. 4  Residual error distributions of object positions of Arc 2
5. CONCLUSIONS

Because of the particularity of space object observations, there exist in their measured CCD images many factors which lead to image degradation, such as the image blurring caused by the trembling during the rapid motion of the telescope, the undersaturated image created by a short exposure time, the differences of background readings made by the image multi-channel readout, and the smears brought about by privation of shutter, etc. All these factors affect the object imaging and degrade the image quality, so extra difficulties are brought in the data processing. In this paper, an image restoration procedure is used on the basis of the maximum entropy estimation, namely, by means of the measurement and statistics of the FWHMs of object images, a two-dimensional Gauss function with the analogous size of FWHM is given, and taken as the PSF to join in the image restoration procedure. In our experiment, two arcs of seriously-degraded GPS satellite optical data are processed, the position coordinates of the object images before and after the treatment are obtained by using Sextractor. Afterwards, a comparison between the equatorial coordinates of the object measured by the astronomical positioning and those calculated from the precise ephemeris is carried out to get the precision of data processing. The experimental result indicates that the measuring errors of object positions before the treatment are greater than $10''$, while after the image restoration the accuracies of the object positions raise substantially, namely, better than $3''$ for the two arcs. The application of the maximum entropy method for image restoration has reduced the influences of image degradation, improved the positioning accuracy of the space object, and our experimental result proves also that to use a two-dimensional Gauss function with an analogous FWHM as the apriori model of image degradation is feasible.

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