On the Extent of the Solar Lower Overshooting Zone†

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Key words: convection—overshooting—solar interior

It is well known that convective overshooting has great impact on the structure and evolution of stars. There is still no satisfactory theory of convective overshooting. Although great advances have been achieved in this field[1–4], the phenomenological non-local mixing-length theory is still the most usual means for treating convective overshooting in the calculation of stellar evolution[5,6]. Helioseismology provides an effective means for probing the structure of solar convection zone. It imposes a stringent constraint on the existing theory of stellar convection.

In the past few years, the probing of the overshooting zone below the solar convection zone by means of helioseismology has attracted great interest. All the theoretical works were based on a simple non-local phenomenological model of overshooting[5–9]. Adjacent to the convectively unstable zone, where the temperature gradient $V$ is close to and slightly higher than the adiabatic value ($V > V_{ad}$), there is an overshooting zone where the temperature gradient is near and slightly less than the adiabatic value ($V < V_{ad}$). Passing through a very thin transition layer, the nearly adiabatic temperature gradient jumps to the radiative one, and at the same time the convective velocity becomes zero. Therefore, a near discontinuity in the derivatives of the sound speed is built up at the base of the overshooting zone. Such a discontinuity will reflect incident acoustic waves and then induce an oscillatory component in the frequencies of solar $p$-modes as a function of the radial order $n^{[10]}$. The amplitude of the oscillatory component depends on the severity of the discontinuity, which in turn depends on the overshooting distance. According to this principle, Gough and Sekii[11] analyzed observational data of solar oscillations, and they concluded that there is no convincing evidence for overshooting. Other authors supposed that the overshooting zone must be extremely narrow. As may be found in the literature, the upper limit is in the range

† Supported by National Natural Science Foundation and National Key Project
Received 2001–01–28; revised version 2001–05–08

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PII: S0275-1062(01)00074-1
Convective overshooting is a non-local phenomenon, which cannot be correctly described with the current phenomenological non-local mixing-length theory of convection. The phenomenological theory is not based on hydrodynamics, but is a simple ballistic description. Such an oversimplified picture leads to many misunderstandings about overshooting. In the present work, a non-local convection model for solar envelope is presented by using a non-local dynamic theory of auto- and cross-correlations of turbulent velocity and temperature.

In Fig. 1, the variations of $\nabla_{rad}$, $\nabla$ and $\nabla - \nabla_{ad}$ as functions of radius near the bottom of the convection zone in our non-local model are shown. From Fig. 1 one can see that the temperature gradient has become already sub-adiabatic ($\nabla < \nabla_{ad}$) in the lower part of the convection zone where $L_c > 0$. In the overshooting zone, however, the temperature gradient is still sub-adiabatic, but it is super-radiative! Namely, $\nabla_{rad} < \nabla < \nabla_{ad}$. Going further, $\nabla$ gradually approaches the radiative value. The extension of the super-radiative region is about $0.63H_p$.

In the regime of our hydrodynamic theory of non-local convection, there is no near discontinuity in the derivatives of sound speed at the bottom of the lower overshooting zone; rather, the temperature gradient turns smoothly from nearly adiabatic stratification to radiative one. Therefore, we should re-examine the conclusions about the size of the overshooting zone deduced from measurements of the discontinuity in the derivatives of sound...
speed using helioseismic observations\textsuperscript{11–15}. The statement that no significant discontinuity in the derivatives of sound speed has been found at the base of the solar convection zone does not mean that there is no overshooting\textsuperscript{11}, nor that the overshooting zone is extremely narrow\textsuperscript{15}. Our results can naturally reconcile the conflict between the tiny overshooting zone from helioseismic observations mentioned above and the mild overshooting distance needed by the solar lithium abundance and overall properties of stellar evolution.

This misunderstanding of the structure of overshooting zone caused by the phenomenological mixing-length theory was pointed out long ago by Xiong\textsuperscript{16} and Petrovay & Marik\textsuperscript{18}. In the phenomenological non-local theory of convection, an implicit assumption, i.e. the turbulent velocity and temperature are completely correlative, seems to be normally included. However, this is not true. The results of our complete non-local theory of correlation functions show that the correlation coefficient of turbulent velocity and temperature decreases very quickly towards the lower boundary of convection zone, and on crossing the boundary of the convection zone the correlation coefficient changes its sign. Kupka\textsuperscript{19} got a similar result by using his non-local convection model. More complete 2D or 3D numerical simulations of compressible convection\textsuperscript{20} also show that in the lower overshooting zone, the convective energy flux becomes negative.

Fig. 2 presents the relative differences in the squared sound speed and density between the local and non-local models. The differences shown in Fig. 2 are very similar to the results of the relative squared sound speed and density differences between the Sun and Model S (SSM) of Christensen-Dalsgaard et al. shown in Figs. 1a and 1b of Basu’s paper\textsuperscript{15}. Therefore, it can be expected that if our non-local convection model is used to replace the SSM of Christensen-Dalsgaard et al. as the reference model of inversion, the relative squared sound speed and density differences mentioned above would be removed, at least they would be greatly reduced. It is not necessary to assume a sudden increase of the opacity at the bottom of convection zone\textsuperscript{21}, or to introduce an unknown mixing below the convection zone\textsuperscript{22}.

References

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