A new equation of state for dark energy model

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
JCAP11(2011)034
(http://iopscience.iop.org/1475-7516/2011/11/034)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 114.212.244.8
The article was downloaded on 02/08/2013 at 03:58

Please note that terms and conditions apply.
A new equation of state for dark energy model

Lei Feng\textsuperscript{a,c} and Tan Lu\textsuperscript{b,c}

\textsuperscript{a}Department of Physics, Nanjing University, Nanjing 210093, China
\textsuperscript{b}Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
\textsuperscript{c}Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University, Purple Mountain Observatory, Nanjing 210093, China

E-mail: fenglei@chenwang.nju.edu.cn, t.lu@pmo.ac.cn

Received September 7, 2011
Revised September 21, 2011
Accepted November 6, 2011
Published November 17, 2011

Abstract. A new parameterization for the dark energy equation of state (EoS) is proposed and some of its cosmological consequences are also investigated. This new parameterization is the modification of Efstathiou’ dark energy EoS parameterization. $w(z)$ is a well behaved function for $z \gg 1$ and has same behavior in $z$ at low redshifts with Efstathiou’ parameterization. In this parameterization there are two free parameter $w_0$ and $w_a$. We discuss the constraints on this model’s parameters from current observational data. The best fit values of the cosmological parameters with 1σ confidence-level regions are: $\Omega_m = 0.2735^{+0.0171}_{-0.0163}$, $w_0 = -1.0537^{+0.1432}_{-0.1511}$ and $w_a = 0.2738^{+0.8018}_{-0.8288}$.

Keywords: dark energy theory, dark energy experiments

\copyright 2011 IOP Publishing Ltd and SISSA

doi:10.1088/1475-7516/2011/11/034
1 Introduction

In recent years, the discovery of accelerating expansion of the universe is an amazing development. It was firstly discovered by observing type Ia supernova (SNe Ia) [1, 2], which can be used as standard candles [3, 4]. The cosmic microwave background (CMB) measurements from Wilkinson Microwave Anisotropy Probe (WMAP) [5] and the large scale structure survey by Sloan Digital Sky Survey (SDSS) [6, 7] confirm this accelerating expansion universe model. There are two kinds of ideas, i.e. the existence of the dark energy or modifications of the gravity theory, to explain this concept. The first scheme is most popularly discussed, and many models have been proposed, such as the holographic dark energy models [8] and the Chaplygin Gas [9]. In addition there are also many modified gravity models, such as the brane world [10] and $f(R)$ [11] and so on.

Following the idea of dark energy, the dark energy EoS which is the ratio of its pressure to its energy density ($w(z) = p/\rho$), has become one of the most remarkable quantities in theoretical and observational cosmology. If $w$ equals $-1$, dark energy is the cosmological constant (LCDM), and it may be the vacuum energy of all the quantum field in the universe. Otherwise the dark energy maybe a dynamical scalar field, such as the Quintessence [12], the quintom [13].

There are at least three popular way to explore the behavior of dark energy EoS. The first one is model-dependent way, such as the solving scalar field equation of a particular theory. Another possibility is to build a functional form for EoS in terms of some free parameters. This is the most popular way, and lots of EoS parameterizations have been discussed in the literature (such as [14–24] and refs. therein). The third approach is picking a simple local basis representation for $w(z)$ (bins, wavelets), and estimate the associated coefficients [25–27]. In addition, there are also some nonparametric way [28, 29].

In this paper, we follow the second approach discussed above and consider a new parameterization for the dark energy EoS. In [20], the author developed a new dark energy EoS parameterization: $w(z) = w_0 + w_a \ln(1 + z)$. It is obvious that when $z \rightarrow \infty$, $w(z)$ has poor behavior and becomes infinite. This dark energy EoS parameterization can only describe the behavior of dark energy when $z$ is not very large. To avoid this problem, we
consider a new dark energy EoS parameterization, which is the modification of Efstathiou’
parameterization: \( w(z) = w_0 + w_a \ln(1 + \frac{z}{1+z}) \). The value of \( w(z) \) is \( w_0 \) at present and \( w(z) \)
becomes to \( (w_0 + w_a * \ln 2) \) when \( z \to \infty \). In this model there are three parameters in all,
which is \( w_0 \), \( w_a \) and \( \Omega_m \). As shown in [30], this EoS will get to a nonphysical value in the
far future time when redshift \( z \) approaches \(-1\), namely, \( |w(z)| \) will grow rapidly and diverge.

In this paper, we perform a global data fitting analysis on this new dark energy EoS
parametrizations, and present constraints on the model parameters from the current obser-
vational data, including the seven-year WMAP data, Baryon Acoustic Oscillations (BAO)
data, Observational Hubble data and SN Union2 sample. Since dark energy parameters are
tightly correlated to some other cosmological parameters, such as the matter density param-
eter \( \Omega_m \) and the Hubble constant \( H_0 \), it is necessary to consider a global fit procedure in the
investigation of the dynamical dark energy. The paper is organized as follows: In section II,
we review the new dark energy EoS parameterization. In section III, we describe Current
Observational Data we used. In section IV, we perform the cosmic observation constraint,
the results are also presented. The last section is the conclusion.

2 New parameterization

Let us start by presenting some of the most investigated EoS parameterizations(see also [22]
for other parameterizations):

\[
w(z) = \begin{cases} 
  w_0 + w_a z & \text{(redshift)} \quad [15–17] \\
  w_0 + w_a z/(1 + z) & \text{(scale factor)} \quad [18, 19] \\
  w_0 + w_a \ln(1 + z) & \text{(logarithmic)} \quad [20]
\end{cases}
\]

where \( w_0 \) is the current value of the EoS, and \( w_a \) are free parameters quantifying the time-
dependence of the dark energy EoS. These parameter must be adjusted by the observational
data. Note that when \( w_a = 0 \) and \( w_0 = -1 \), the dark energy model reduce to LCDM model.

The first parameterization represents a good fit for low redshifts, but has serious prob-
lems to explain high-\( z \) observations since it blows up at \( z > 1 \) as \( \exp(3w_a z) \) for values of
\( w_a > 0 \). For example, it can not explain the estimated ages of high-\( z \) objects [31]. The second
one solves this problem, since \( w(z) \) is a well behaved function for \( z \gg 1 \) and recovers the
linear behavior in \( z \) at low redshifts. The latter was introduced by Efstathiou [20]. It was
built empirically to adjust some quintessence models at \( z \lesssim 4 \). When \( z \) approaches infinity,
\( w(z) \) has poor behavior and becomes infinite and this is unnatural. Similar to the second
model, let us consider the following EoS parameterization

\[
w(z) = w_0 + w_a \ln \left( \frac{1 + \frac{z}{1+z}}{1} \right). \tag{2.1}
\]

where the value of \( w(z) \) is \( w_0 \) at present and \( w(z) \to (w_0 + w_a * \ln 2) \) when \( z \to \infty \).

If there is no interaction between dark energy and other component of the universe, one
can show from the energy conservation law \[ \dot{\rho} = -3 \dot{a} (\rho + p)/a \] that the dark energy density evolves as

\[
\rho_{DE}(z) = \rho_c (1 - \Omega_m) \exp \left( 3 \int_0^z \frac{dz'}{1 + w(z')} \right) . \tag{2.2}
\]
where $\rho_c$ is the critical density, it is defined by the following equation

$$
\rho_c = \frac{3H_0^2}{8\pi G_N}.
$$

(2.3)

In this new parameterization, it is hard to write out the analysis formula of this quantity, It is calculated through numerical method.

In order to study the evolution of cosmological perturbations, we use the public Parameterized Post-Friedmann (PPF) package developed by Wayne Hu(see e.g. [32] for detailed discussions on PPF method) to calculate the CMB anisotropy spectrum. In figure 1 we plot the anisotropy spectrum for different choices of $w_0$ and $w_a$. We observe some obvious differences with respect to the LCDM case on larger scales (multipole number $l < 10$). And for the model $w_0 = -0.7$ and $w_a = -0.2$, there is a split deviation on the peak of anisotropy spectrum.

3 Current observational data

In order to test this new model, we use the most recent observational data currently available. In this section, we describe how we use these data.

3.1 Type Ia supernovae constraints

We use the 557 SNe Ia Union2 dataset [33]. Following [34, 35], one can obtain the corresponding constraints by fitting the distance modulus $\mu(z)$ as

$$
\mu_{\text{th}}(z) = 5 \log_{10}[D_L(z)] + \mu_0.
$$

(3.1)

where $\mu_0 = 42.38 - 5 \log_{10} h$, and $h$ is the Hubble constant $H_0$ in units of 100 km s$^{-1}$ Mpc$^{-1}$, In flat universe, the Hubble-free luminosity distance $D_L = H_0 d_L$ is

$$
D_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z')},
$$

(3.2)

where $E(z) \equiv H(z)/H_0$. 

Figure 1. the CMB anisotropy spectrum. Red line: $w_0 = -0.7$, $w_a = -0.2$; Green line: $w_0 = -1.2$, $w_a = 0.2$; Black line: $w_0 = -1$, $w_a = 0$. 

-3-
For the SN Ia dataset, the best fit values of the parameters can be determined by a likelihood analysis, based on the calculation of

$$\chi^2_{\text{SN}} = \sum \frac{[\mu_{\text{th}}(z_i) - \mu_{\text{obs}}(z_i)]^2}{\sigma^2(z_i)} \quad (3.3)$$

3.2 Baryon Acoustic Oscillation constraints

The BAO data come from SDSS DR7 \[36\]. The datapoints we use are

$$\frac{r_s(z_d)}{D_V(0.275)} = 0.1390 \pm 0.0037 \quad (3.4)$$

and

$$\frac{D_V(0.35)}{D_V(0.2)} = 1.736 \pm 0.065, \quad (3.5)$$

where the spherical average gives us the following effective distance measure \[37\],

$$D_V(z) = \left[ \left( \int_0^z \frac{dx}{H(x)} \right)^2 \frac{z}{H(z)} \right]^{1/3} \quad (3.6)$$

and \(r_s(z_d)\) is the comoving sound horizon at the baryon drag epoch. Also, \(z_d\) can be obtained by using a fitting formula \[38\] :

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}], \quad (3.7)$$

with

$$b_1 = 0.313(\Omega_m h^2)^{-0.419} [1 + 0.607(\Omega_m h^2)^{0.674}], \quad (3.8)$$

$$b_2 = 0.238(\Omega_m h^2)^{0.223}. \quad (3.9)$$

The function \(r_s(z)\) is the comoving sound horizon size

$$r_s(z) = \frac{c}{\sqrt{3}} \int_0^{1/(1+z)} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_b/(4\Omega_\gamma) a)^2}} \quad (3.10)$$

where \(\Omega_\gamma = 2.469 \times 10^{-5} h^{-2}\) for \(T_{\text{CMB}} = 2.725 K\).

So the \(\chi^2\) for the BAO data is given by

$$\chi^2_{BAO} = \left( \frac{r_s(z_d)/D_V(z = 0.275) - 0.1390}{0.0037} \right)^2$$

$$+ \left( \frac{D_V(z = 0.35)/D_V(z = 0.2) - 1.736}{0.065} \right)^2. \quad (3.11)$$
3.3 Cosmic Microwave Background constraints

The CMB shift parameter $R$ is provided by [39]

$$ R(z_{\text{rec}}) = \frac{\sqrt{\Omega_m H_0^2}}{\sqrt{|\Omega_k|}} \sinh \left[ \sqrt{|\Omega_k|} \int_0^{z_{\text{rec}}} \frac{dz'}{H(z')} \right], $$

(3.12)

where $\sinh(x)$ is $\sinh(x)$ for $\Omega_k > 0$, $x$ for $\Omega_k = 0$, and $\sin(x)$ for $\Omega_k < 0$, respectively. Here, the redshift $z_{\text{rec}}$ (the decoupling epoch of photons) is obtained by using the fitting function [40]

$$ z_{\text{rec}} = 1048 \left[ 1 + 0.00124(\Omega_b h^2)^{-0.738} \right] \left[ 1 + g_1(\Omega_m h^2)^{g_2} \right], $$

where

$$ g_1 = 0.0783(\Omega_b h^2)^{-0.238} \left( 1 + 39.5(\Omega_b h^2)^{0.763} \right)^{-1}, $$

$$ g_2 = 0.560 \left( 1 + 21.1(\Omega_b h^2)^{1.81} \right)^{-1}. $$

In addition, the acoustic scale is related to the distance ratio and is expressed as

$$ l_A = \frac{\pi}{r_s(z_{\text{rec}})} \frac{c}{\sqrt{|\Omega_k|}} \sinh \left[ \sqrt{|\Omega_k|} \int_0^{z_{\text{rec}}} \frac{dz'}{H(z')} \right]. $$

(3.13)

Following ref. [41], the $\chi^2$ for the CMB data is

$$ \chi^2_{\text{CMB}} = \sum_{j=1}^{12} \frac{(H(z_j) - H_{\text{obs}}(z_j))^2}{\sigma_{H,j}^2}, $$

(3.14)

where $x_i = (l_A, R, z_{\text{rec}})$ is a vector and $(C^{-1})_{ij}$ is the inverse covariance matrix. The seven-year WMAP observations [41] give the maximum likelihood values: $l_A(z_{\text{rec}}) = 302.09$, $R(z_{\text{rec}}) = 1.725$ and $z_{\text{rec}} = 1091.3$. In ref. [41], the inverse covariance matrix is also given as follows

$$ (C^{-1}) = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.270 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix}. $$

(3.15)

3.4 Observational Hubble Data (OHD)

The Hubble parameter can be written as the following form:

$$ H(z) = -\frac{1}{1+z} \frac{dz}{dt}. $$

(3.16)

So, through measuring $dt/dz$, we can obtain $H(z)$. In [42] and [43, 44], the author indicated that it is possible to use absolute ages of passively evolving galaxies to compute values of $dt/dz$. The galaxy spectral data used by [44] come from the Gemini Deep Deep Survey [45] and archival data [46–51]. Detailed calculations of $dt/dz$ can be found in [44], so we do not discuss them here. Currently, we have a set of 12 values of the Hubble parameter versus redshift in total (see table 2 of [52]). A particularly attractive feature of this test is that differential ages are less sensitive to systematic errors than absolute ages [53].

We can use these data to constrain different kinds of dark energy models and modified gravity models by minimizing the quantity

$$ \chi^2_{\text{OHD}} = \sum_{j=1}^{12} \frac{(H(z_j) - H_{\text{obs}}(z_j))^2}{\sigma_{H,j}^2}. $$

(3.17)

This test has already been used to constrain several cosmological models [54–67].
4 Results

In our analysis, we perform a global fitting to determine the cosmological parameters using the MCMC method. In our calculations, we have taken the total likelihood function \( L \propto e^{-\chi^2/2} \) to be the products of the separate likelihoods of SNe Ia, BAO, CMB and OHD. Then we get \( \chi^2 \) as

\[
\chi^2 = \chi^2_{SN} + \chi^2_{BAO} + \chi^2_{CMB} + \chi^2_{OHD},
\]

(4.1)

The results on the best fit values of the cosmological parameters with 1\( \sigma \) confidence-level regions are: \( \Omega_m = 0.2735^{+0.0171}_{-0.0163} \), \( w_0 = -1.0537^{+0.1432}_{-0.1511} \) and \( w_a = 0.2738^{+0.0818}_{-0.0828} \). The nuisance parameters \( H_0 \) used in the analysis is actually not model parameters with significant meanings, so we do not list it.

In figures 2, we also show the parametric spaces \( w_0 - w_a \) and \( \Omega_m - w_0 \) that arise from the joint analysis described above. We note that the result is consistent with the LCDM (\( w_0 = -1 \) and \( w_a = 0 \)) model in the 1\( \sigma \) CL. To acquire more information on the property of dark energy, in figure 3 we plot the evolution of the EoS \( w(z) \) along with \( z \). The following is some discussion of these findings:

1. The best-fit results are: \( \Omega_m = 0.2735 \), \( w_0 = -1.0537 \) and \( w_a = 0.2738 \). Note that here the results are maximum likelihood values. The value of \( w(z) \) is \(-1.0537\) at present and \( w(z) \) equals to \(-0.8639\) when \( z \to \infty \). We find that the best-fit dark energy model is a quintom model [13], whose \( w(z) \) crosses the cosmological constant boundary \( w = -1 \) during the evolution. And the redshift is \( 0.2766 \) when \( w(z) \) crosses the cosmological constant boundary \( w = -1 \).

2. With the current observational data, the variance of \( w_0 \) and \( w_a \) we get are still large; the 1\( \sigma \) constraints on \( w_0 \) and \( w_a \) are \( w_0 = -1.0537^{+0.1432}_{-0.1511} \) and \( w_a = 0.2738^{+0.0818}_{-0.0828} \). This result implies that though the dynamical dark energy models are mildly favored, the current data cannot distinguish different dark energy models decisively. With the fitting results we obtained, we can reconstruct the evolution of the EoS of dark energy, \( w(z) \) which is shown in figure 3. From the figure, we can directly see that although the quintom model is more favored, LCDM, however, still cannot be excluded.
Figure 3. The evolution of $w(z)$ along with $z$ for the model considered in this manuscript. The result is consistent with the cosmological constant in the $1\sigma$ CL.

3. From figure 3, one can see that the allowed value of $w(z)$ is in the band $\{-1.7, -0.4\}$, which is relatively narrow.

4. The best fit value and $1\sigma$ confidence-level regions of $\Omega_m$ is $0.2735^{+0.0171}_{-0.0163}$, which is also consistent with the constraint on $\Omega_m$ in the LCDM model and the CPL model.

5 Conclusion

In this paper, we develop a new parameterization which is the modification of Efstathiou’ parameterization. In this new parameterization, there are three free parameters: $w_0$, $w_a$ and $\Omega_m$. Then we carry out the global fitting on these model using the current data: SNe Ia, BAO, CMB and OHD. From the analysis, the best fit values of the cosmological parameters with $1\sigma$ confidence-level regions are: $\Omega_m = 0.2735^{+0.0171}_{-0.0163}$, $w_0 = -1.0537^{+0.1432}_{-0.1511}$ and $w_a = 0.2738^{+0.8018}_{-0.8288}$. From the analysis, we can directly see that the quintom model is more favored, but this result is also consistent with the LCDM model in the $1\sigma$ CL.

Acknowledgments

Thank Xiaodong Li for helping FL write some figure of this manuscript.

References


[5] WMAP collaboration, D. Spergel et al., First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: determination of cosmological parameters, 

[6] SDSS collaboration, M. Tegmark et al., The 3D power spectrum of galaxies from the SDSS, 

[7] M. Tegmark et al., Cosmological parameters from SDSS and WMAP, 

[8] K. Ke and M. Li, Cardy-Verlinde formula and holographic dark energy, 


J. Phys. A 16 (1983) 2757 [inSPIRE]; J.D. Barrow and S. Cotsakis, Inflation and the conformal structure of higher order gravity theories, 


[12] B. Feng, X.-L. Wang and X.-M. Zhang, Dark energy constraints from the cosmic age and supernova, 

[13] Y. Wang and M. Tegmark, New dark energy constraints from supernovae, microwave background and galaxy clustering, 

[14] R. Cooray and D. Huterer, Gravitational lensing as a probe of quintessence, 

[15] P. Astier, Can luminosity distance measurements probe the equation of state of dark energy, 

[16] J. Weller and A. Albrecht, Opportunities for future supernova studies of cosmic acceleration, 

[17] M. Chevallier and D. Polarski, Accelerating universes with scaling dark matter, 

[18] E.V. Linder, Exploring the expansion history of the universe, 

[19] G. Efstathiou, Constraining the equation of state of the Universe from distant Type Ia supernovae and cosmic microwave background anisotropies, 

[20] M. Goliath et al., A unified equation of state of dense matter and neutron star structure, 


